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Firm-specific attributes and the cross-section of momentum ☆

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Abstract

This paper identifies observable firm-specific attributes that drive momentum. We find that a firm's revenues, costs, and growth options combine to determine the dynamics of its return autocorrelation. We use these insights to implement momentum strategies (buying winners and selling losers) with both numerically simulated returns and CRSP/Compustat data. In both sets of data, momentum strategies that use firms with high revenue growth volatility, low costs, and valuable growth options outperform traditional momentum strategies by approximately 5% per year.

JEL classification: G12; G14

Keywords: Asset pricing; Expected returns; Momentum; Real options

1. Introduction

This paper studies firms that exhibit momentum and firms that do not. We ask two questions: (i) Do-firm specific attributes (revenues, costs, and real options) affect the ability

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of past returns to predict future expected returns?; and (ii) How can firm-specific attributes be used to create "enhanced momentum strategies"? In our paper, an enhanced momentum strategy entails buying specific winners and selling specific losers in such a manner as to produce larger profits than strategies documented in Jegadeesh and Titman (1993). Our work is motivated by the vast momentum research conducted over the past decade. For instance, momentum has been linked to market-related variables such as a firm's share turnover, liquidity, book-to-market ratio, analyst coverage, aggregate market conditions, as well as to attributes such as a firm's industry, earnings, and growth. In addition, momentum has been confirmed in both the U.S. and overseas markets.[1]

The profitability of momentum strategies is a cross-sectional result: winners realize higher average returns than losers. Suppose one can identify firms with time-varying return autocorrelation and can restrict a momentum strategy to those firms whose autocorrelation is conditionally higher than average. Ceteris paribus, this restricted strategy results in enhanced profits because winners (losers) with relatively high autocorrelated returns have more persistent expected returns than winners (losers) from an unrestricted strategy.

To study enhanced momentum strategies, we model a single firm that has realistic attributes such as revenues, costs, growth options, and shutdown options. Our analysis makes two contributions. First, we link time-varying return autocorrelation to firm attributes and relate momentum profits to the same firm attributes. Second, our model explains both qualitatively and quantitatively empirical evidence regarding momentum that has previously been viewed as anomalous.

A prominent stylized fact is that firms with high market-to-book ratios produce enhanced momentum profits, e.g., Asness (1997). Daniel and Titman (1999) argue that such profitability stems from the large weight of intangible assets in high market-to-book firms and the fact that investors overreact to news related to intangible assets. In our model, firms with valuable growth options exhibit higher return autocorrelation than firms without such growth options. The rationale is as follows: Firms that performed well in the recent past are better poised to exploit their growth options. Because these options are risky assets that now account for a larger fraction of firm value, such firms are riskier. In turn, they are associated with higher expected returns. This effect is only temporary, however, as firms eventually use or lose their growth options. We use a firm's market-to-book ratio as a proxy for the presence of growth options. In both numerically simulated firms and CRSP/Compustat firms, high market-to-book firms produce approximately 10% higher momentum profits per annum than low market-to-book firms.

We document that low cost of goods sold (CGS) firms produce enhanced momentum profits 2% to 9% higher per annum than high CGS firms. In our model, costs effectively introduce leverage, which can lower return autocorrelation to the point of becoming negative. A positive shock to revenues in a firm with fixed costs leads to higher profit margins and a higher stock price today. The associated decrease in risk, and hence, in

[1]Some of the many works include Bernard and Thomas (1989), Jegadeesh and Titman (1993, 1995, 2001a,b, 2002), Chan, Jegadeesh, and Lakonishok (1996, 1999), Asness (1997), Rouwenhorst (1998), Daniel and Titman (1999, 2006), Moskowitz and Grinblatt (1999), Lee and Swaminathan (2000), Grundy and Martin (2001), Hong and Stein (1999), Hong, Lim, and Stein (2000), Bhojraj and Swaminathan (2004), Chordia and Shivakumar (2002), Cooper, Gutierrez, and Hameed (2002), Lewellen (2002), Sadka (2004), George and Huang (2004), Griffin, Ji, and Martin (2003), and Liu, Warner, and Zhang (2003). A search for the word "momentum" in a paper's title or abstract yields 500 citations from SSRN and 454 citations from EconLit.

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expected returns, is more pronounced in low margin (high cost) firms than in high-margin firms.

We also document that high revenue volatility firms produce momentum profits 6% to 14% higher per annum than low revenue volatility firms. Behavioral explanations equate volatility with information uncertainty surrounding the firm. The uncertainty exacerbates investor overconfidence in stocks that may be hard to sell short (Jiang, Lee, and Zhang, 2005). In our model, high revenue volatility firms have more dispersion in expected returns than low revenue volatility firms. In particular, past winners have higher expected returns, on average, than past losers, and this disparity is higher in a sample of high revenue volatility firms.

Finally, Cooper, Gutierrez, and Hameed (2004) find quarterly momentum returns of 2.82% in up markets and −1.11% in down markets. Our simulated panel of firms also produces higher quarterly momentum profits in up markets (3.09%) than down markets (0.20%). During up markets, firms tend to move closer to exercising their growth options, which tends to increase return autocorrelations. During down markets, firms tend to move closer to financial distress, which tends to decrease return autocorrelations.

In order to better understand the economic intuition behind these findings, we now provide two sets of examples. These examples also help to motivate our formal model in Section 2.

1.1. Intuition relating to dynamic return autocorrelation

A simple, two-asset portfolio helps to illustrate the role of return autocorrelation. Suppose the portfolio contains a risky stock with a positive risk premium and a risk-free asset. What happens to the future riskiness of the portfolio if the portfolio's realized excess returns are "good" today (i.e., above average)? The answer to this question depends on whether the portfolio consists of long or short positions.[2] The chart below summarizes the two most common weighting schemes. Scheme 1 exhibits positive return autocorrelation while Scheme 2 exhibits negative return autocorrelation:

Scheme	Risky Asset	Risky-free Asset	Return Autocorrelation
1	long	long	+
2	long	short	−

In Scheme 1, an increase in the stock price leads to a higher weighting of the risky asset (the stock) in the portfolio, higher overall systematic risk, and higher expected returns. Good news is followed by higher risk in this scheme. In Scheme 2, an increase in the stock price leads to lower leverage and thus lower systematic risk for the portfolio. Note that this analysis does not rely on the presence of a risk-free asset; we could also consider a portfolio with one risky asset and one less risky asset.

[2] A portfolio cannot be short both assets since weights must add to one. A third scheme, short the risky asset and long the risk-free asset, also exhibits positive return autocorrelation, but it does not apply to most industrial firms. A related two-asset portfolio analysis is found in Rubinstein (1983) in the context of deriving option prices.

The intuition behind the theoretical model in Section 2 comes from treating the firms as a portfolio of assets. These assets consist of revenues from existing capital, costs due to operations, growth options, and limited liability (shutdown) options. We examine how these different factors affect return autocorrelation. The simple portfolio example above suggests that combining revenues from existing capital with costs can lead to negative return autocorrelation, because such firms are long a risky asset and short a less risky asset, as in Scheme 2. On the other hand, adding growth options to existing capital is akin to being long both a risky asset (the growth option) and a less risky asset (existing capital). In this case, Scheme 1 suggests the presence of positive return autocorrelation. Finally, a limited liability option, as in the case of portfolio insurance, can reduce risk in the event that the firm performs poorly, thereby increasing return autocorrelation. Thus, one can see that intuitively at least, conditional return autocorrelation is a time-varying function of the relative weights of current revenues, costs, growth options, and the limited liability option of the firm. Section 2 clarifies the contribution of each of these factors to return autocorrelation.

Example. Consider a drug manufacturer that is currently marketing a drug. The demand for this drug is well known, therefore cash flows are fairly steady and the riskiness of the firm is low. The firm's value derives entirely from the present value of these cash flows. The firm begins development of a second drug whose future demand is uncertain. The decision to ultimately market the second drug is contingent on demand for the second drug. In early stages of development, it is very unlikely the second drug will make it to market. The second drug therefore contributes little to the overall value of the firm and news about potential future demand for the second drug has little effect on the firm's value. If the second drug makes it to advanced stages of development, the value of the overall firm increases because cash flows from marketing the second drug are now more likely. Reaching advanced stages also means that potential cash flows from the second drug constitute a higher fraction of overall firm value. Accordingly, firm value is significantly more sensitive to news about future demand for the second drug. As long as news about demand for the second drug has a systematic component, expected returns increase along with firm value—i.e., there will be positive return autocorrelation.

1.2. Intuition relating to enhanced momentum profits

Fig. 1 provides additional intuition relating to the sources of return autocorrelation. Panel A plots the log value of two simulated firms against the log price of the good they produce. In this example, both firms have fixed production and the graphs are the result of our model from Section 2. Section 3 provides details about the analysis that underlies Fig. 1. The "pre-exercise" firm has a valuable growth option. The "post-exercise" firm has exercised its option and is currently producing more units of the output good than the pre-exercise firm. Regardless of current production levels, as the price of the output good goes up, the value of each firm also goes up. Panel A shows this explicitly as both graphs increase monotonically.

More important, Panel B of Fig. 1 shows the sensitivity of firm log value to changes in the log price of the output good. We refer to this sensitivity as the "factor loading" or "beta" of the firm because it represents a measure of compensated risk in our model. For the pre-exercise firm, an increase in output price dramatically increases risk and therefore

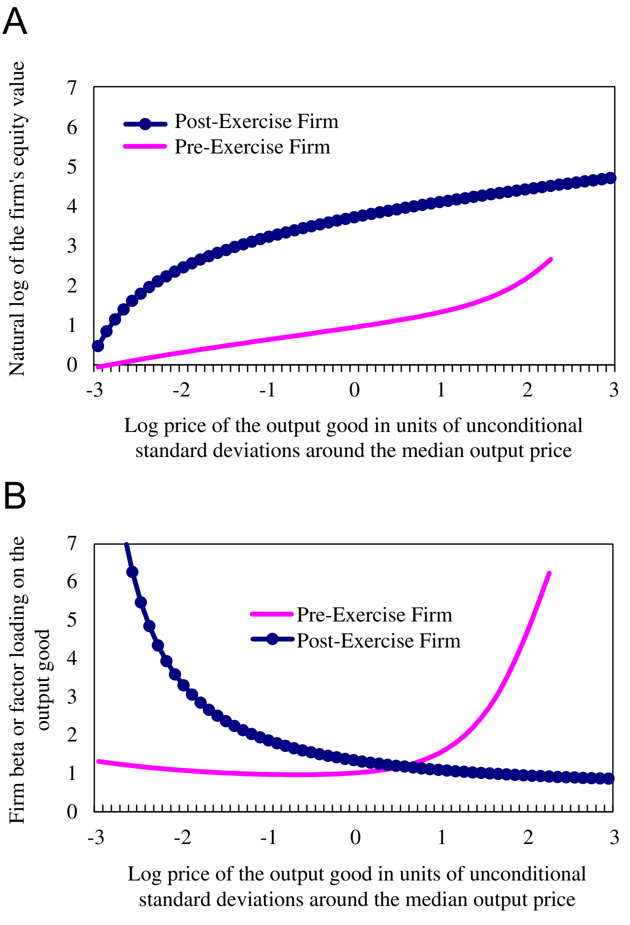

Fig. 1. High-cost model firms. This figure shows the effect of output price on a firm's value and factor loading. We graph two firms that are otherwise identical except one has a growth option and one has exercised its growth option. Panel A shows the firm's log value as a function of the log price of the output good. Panel B shows the beta (factor sensitivity) of the firm as a function of the log price of the output good. The firms have high costs relative to the price of the output good. Parameter values and a description of the numerical analysis are given in the text.

expected returns. Thus, an increase in the pre-exercise firm's stock price today is followed by an additional increase in returns (on average) over the near to medium term. The plot depicts positive return autocorrelation, which is present due to the pre-exercise firm's large growth option.

For the post-exercise firm, Panel B shows that a drop in the output price of the good increases the firm's factor loading. The increase in factor loading (risk) is due to the well-known leverage effect. Thus, a decrease in the post-exercise firm's stock price today is followed by an increase in returns (on average) over the near to medium term. This describes negative return autocorrelation. The leverage effect for the pre-exercise firm is overwhelmed by the presence of the growth option over the region shown.

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Our model and figures also help illustrate the profitability of momentum strategies. Fig. 2 shows two firms that are much like the simulated firms in Fig. 1 except that these firms have lower costs. Ceteris paribus, winners are more likely to come from the right side of the axes in Fig. 2 and losers are more likely to come from the left side. Panel B shows that winners have higher average factor loadings than losers (a weighted average of the factor loadings at a log price of +1 is higher than a weighted average at a log price of −1). The difference in average factor loadings between the right and left sides of Panel B leads to the prediction that low cost firms generate momentum strategy profits. In contrast, the profitability of momentum trading strategies restricted to high cost firms (see Fig. 1) is by no means assured. Although winners have high average factor loadings (the right side of Fig. 1, Panel B), losers also have higher than average factor loadings (the left side of Fig. 1,

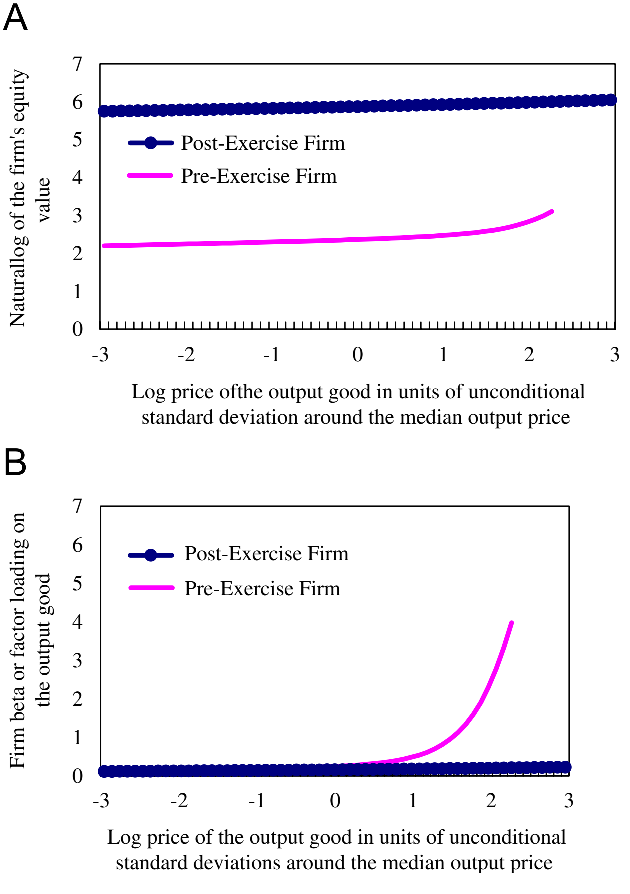

Fig. 2. Low-cost model firms. This figure shows the effect of output price on a firm's value and factor loading. We graph two firms that are otherwise identical except one has a growth option and one has exercised its growth option. Panel A shows the firm's log value as a function of the log price of the output good. Panel B shows the beta (factor sensitivity) of the firm as a function of the log price of the output good. The firms have low costs relative to the price of the output good. Parameter values and a description of the numerical analysis are given in the text.

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Panel B.) A portfolio of high cost firms that is long winners and short losers may very well be unprofitable.

Finally, Fig. 2, Panel B shows there is dispersion in expected returns as one moves from losers (left side) to winners (right side). Although not shown, this dispersion increases as revenue growth volatility increases. The net result is that we can create enhanced momentum strategies by limiting ourselves to firms with high revenue growth volatility.

1.3. Comparison with recent studies

Our paper builds on recent theoretical work that analyzes the relations between firm-specific attributes and expected returns. Following Berk, Green, and Naik (1999), Johnson (2002), Carlson, Fisher, and Giammarino (2004), and Zhang (2005), we model a single firm in partial equilibrium.[3] We employ a single-firm model as such models can be used to study both time-series and cross-sectional properties of returns. Like Carlson, Fisher, and Giammarino (2004) and Zhang (2005), we consider growth options, costs, and the possibility of negative cash flows. Our paper differs from these two in that it primarily studies momentum and return autocorrelation.

Conrad and Kaul (1998) and Berk, Green, and Naik (1999) are the first to study "rational" momentum strategies. The former paper generates momentum strategy profits by simulating returns of stocks with different betas, where the betas are constant and unrelated to the microeconomics of the firm. The latter paper produces (counterfactual) negative momentum profits at a three-month horizon and positive profits beyond a one-year horizon. Focusing on quarterly profits, our model is capable of roughly matching the standard market, size, value, and momentum excess returns. Because we focus on both return autocorrelation and momentum strategies, we are also able to roughly match the profitability of enhanced momentum strategies.

Johnson (2002) models time-varying growth rates in a firm's dividend process. He shows that a firm whose log value is convex with respect to growth rates exhibits positive return autocorrelation. This observation plays an important role in our model as well. However, our model departs from his by including observable firm attributes in the determinants of cash flows. These attributes allow us to examine the sensitivity of return autocorrelation to a richer set of variables, which in turn guides our choice of the enhanced momentum strategies we test. Berk, Green, and Naik (1999) also produce firms with autocorrelated returns. While Johnson's model firm demonstrates positive return autocorrelation, the firms in Berk, Green, and Naik (1999) exhibit negative autocorrelation. Our model, by contrast, produces firms with time-varying autocorrelation that can be either positive or negative.

A separate line of research suggests that investor behavior is responsible for momentum in stock prices (see, e.g., Daniel, Hirshleifer, and Subrahmanyam, 1998; Daniel and Titman, 1999, 2006; Barberis, Shleifer, and Vishny, 1998; Grinblatt and Han, 2005).[4]

[3] Gomes, Kogan, and Zhang (2003) extend the Berk, Green, and Naik (1999) model to a general equilibrium setting (see also Novy-Marx, 2006). Momentum can also appear in models with asymmetric information (see Biais, Bossaerts, and Spatt, 2005; Strobl, 2006), models that deviate from the rational expectations hypothesis (see Banerjee, Kaniel, and Kremer, 2006), and models with uncertainty over systematic risk exposure (Wang, 2005).

[4] There is also an empirical literature that finds momentum in portfolios of stocks, i.e., the winners and losers are industry or Fama-French portfolios as opposed to individual stocks (see Moskowitz and Grinblatt, 1999; Lewellen, 2002). Lewellen (2002) examines the portfolios used to form these momentum strategies and finds small

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While our paper does not preclude behavioral explanations for our empirical findings, we offer a consistent and intuitive theoretical framework that can be calibrated to the data. Note too that we do not preclude macroeconomic effects in our paper. Financial leverage—a cost to equity holders and an important determinant of return autocorrelation—undoubtedly varies with interest rates. Here, however, we simplify matters by limiting our investigation to time-varying expected returns that result from the microeconomics of the firm.

1.4. Road map of the paper

Readers who are more interested in our empirical results may consider starting with Section 4. In that section, we test the implications of our model. In particular, we present results for three enhanced momentum strategies. Readers can then return to Section 2 and review our model. The model produces all three enhanced momentum strategies. The model also provides the background for understanding why momentum profits are linked to firm-specific attributes.

Alternatively, one may begin with Section 2, where we present our model framework and explore firm-specific attributes that lead to return autocorrelation. Section 3 reports the results of a numerical analysis in which we construct model momentum portfolios. It is in this section that we link the predictions of our model to both observable quantities and empirical tests. Section 4 tests the implications of our model with CRSP/Compustat data. We present enhanced momentum strategies by conditioning on firms that our model predicts exhibit return autocorrelation. Finally, Section 5 concludes.

2. A real options model

Our model firm produces a single good and operates in a price-taking environment. The manager's objective is to maximize the value of the firm for equity holders. Production costs are fixed and we abstract away from the strategic issuance of debt. There are no taxes in our model. The manager has a one-time, irreversible expansion option to invest in a positive net present value project; the option is optimally exercised based on the current price of the output good.[5] The shareholders and the manager enjoy limited liability in the sense that they can walk away from the firm when its present value goes to zero (e.g., Brennan and Schwartz, 1984; Fischer, Heinkel, and Zechner 1989; Leland, 1994). Finally, firms can experience a disastrous Poisson-distributed shock forcing nondiscretionary default. The Poisson shocks help calibrate the exit rate of our firm to the exit rate observed in data.

Our analysis is separated into the following Sections: in Section 2.1, we derive general conditions under which firms exhibit positive return autocorrelation; in Section 2.2, we solve for the value and expected returns of a basic (revenue-only) firm; in Section 2.3, we add costs to the basic firm; and in Section 2.4, we end by adding growth and shutdown

(*footnote continued*)
but significant unconditional negative autocorrelation. We focus on conditional autocorrelation in individual stocks.

[5]Using current profitability leads to the same exercise rule since costs are fixed.

options to our firm. At each step, we analyze the contribution of components to return autocorrelation.

2.1. Conditions leading to return autocorrelation

Consider a firm whose cash flow (c_t^*) is determined by a vector ($\boldsymbol{X}_t$) of N distinct sources of risk (factors). Assume the factors evolve according to an Itô diffusion, where $\boldsymbol{\mu}_t$ is the unadjusted drift of the factors, $\boldsymbol{\mu}_t^*$ is the vector of risk-adjusted drifts, $\boldsymbol{\sigma}_t'$ is the vector of factor volatilities, V_t is the value of the firm, and r_t is the risk-free rate. If V_t depends on time only through its dependence on other dynamic variables such as $\boldsymbol{X}_t$, we will at times refer to it as $V(\boldsymbol{X}_t)$, suppressing the time subscript. Defining $v_t \equiv \ln V_t$, we solve for the expected total rate of return of the firm as follows (see Appendix A for details and proofs):

$$\text{Total Rate of Return}_t = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)' \cdot \frac{\partial v_t}{\partial \boldsymbol{X}_t} + r_t. \tag{1}$$

Each $\partial v_t / \partial X_t^n$, $n = 1, \ldots, N$, in Eq. (1) can be interpreted as one of the firm's factor loadings (or factor betas). These loadings are generally time varying. The term $\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*$ is the vector of risk premia associated with the sources of risk the firm faces. Eq. (1), a restatement of the Arbitrage Pricing Theory (see Ross, 1976), gives the value-weighted sum of returns on the assets in the firm. We say that the firm exhibits *positive return autocorrelation* whenever its expected returns increase with firm log value. The following proposition links return autocorrelation to the curvature/convexity of the firm's log value:

Proposition 1. *Assume that $\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*$ is constant and that v_t is a twice-differentiable function of $\boldsymbol{X}_t$. The instantaneous return autocorrelation (sensitivity of expected returns to a change in firm log value dv_t) is*

$$\frac{(\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)' \cdot \dfrac{\partial^2 v_t}{\partial \boldsymbol{X}_t \partial \boldsymbol{X}_t'} \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t' \cdot \dfrac{\partial v_t}{\partial \boldsymbol{X}_t}}{\dfrac{\partial v_t}{\partial \boldsymbol{X}_t}' \cdot \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t' \cdot \dfrac{\partial v_t}{\partial \boldsymbol{X}_t}}. \tag{2}$$

All proofs are found in Appendix A.2. A firm exhibits positive conditional return autocorrelation whenever the expression in Proposition 1 is strictly positive. Eq. (2) is the slope coefficient from a regression of changes in the firm's expected returns on small unexpected changes in firm log value. This result intimately links the presence of return autocorrelation with the curvature of the log value of the firm (i.e., $\partial^2 v_t / \partial \boldsymbol{X}_t \partial \boldsymbol{X}_t'$). The relation to the curvature of the log value is particularly easy to see if the firm's value only depends on a single source of risk, p_t. In this case, the factor loading or firm beta follows from Eq. (1) and is a measure of riskiness:

$$\beta(p_t) \equiv \frac{\partial v_t}{\partial p_t}. \tag{3}$$

Expression (2) from Proposition 1 must be positive for a firm to exhibit positive return autocorrelation. In the one-factor case in which $\mu_t - \mu_t^*$ is constant, the instantaneous return autocorrelation is

$$\frac{(\mu_t - \mu_t^*)\partial^2 v_t/\partial p_t^2}{\beta(p_t)}, \tag{4a}$$

or equivalently,

$$(\mu_t - \mu_t^*)\frac{\partial}{\partial v_t}\beta(p_t). \tag{4b}$$

If $\mu_t - \mu_t^* > 0$, Eq. (4b) has the intuitive interpretation that the firm exhibits positive return autocorrelation whenever its beta (i.e., $\partial v_t/\partial p_t$) increases with value. In addition, if $\beta(p_t)$ is assumed to be positive, then Eq. (4a) says that firms exhibit positive return autocorrelation if their log value is convex in some underlying risk factor. The latter point is noted by Johnson (2002).

Example. How do Eqs. (4) help us understand return autocorrelation for some asset? Consider an asset whose price, P_t, evolves as a geometric Brownian motion (GBM). The drift of GBM is proportional to the level of P_t and therefore is not constant, but $p_t \equiv \ln(P_t)$ has constant drift and therefore can be used as the factor when applying (4). If the value of a firm is $V_t = P_t$, then the factor sensitivity of V_t is $\beta(p_t) = \partial \ln(V_t)/\partial p_t = \partial \ln(P_t)/\partial p_t = 1$, meaning GBM has constant risk premium and the log convexity is zero, because $\partial^2 \ln(P_t)/\partial p_t^2 = 0$. Thus, GBM does not exhibit return autocorrelation. If $V_t = P_t - K$, where K is constant and positive, then $\partial^2 \ln(P_t - K)/\partial p_t^2 < 0$. In other words, a levered portfolio has negative convexity and exhibits negative return autocorrelation. Finally, let C_t^E be a Black-Scholes call option written on a GBM asset with price P_t. The delta of the call option, $\partial C_t^E/\partial P_t > 0$, increases with P_t. The riskiness of the call option from Eq. (3) is positive, $\partial \ln(C_t^E)/\partial p_t > 0$, but the riskiness *decreases* with P_t. In other words, the call option's log convexity is *negative*, $\partial^2 \ln(C_t^E)/\partial p_t^2 < 0$, and the call option exhibits negative return autocorrelation.

From this point onward, we assume that the firm is affected by a single factor (p_t) and turn to a more systematic investigation of firm-specific attributes and return autocorrelation. We view the firm as a portfolio of assets, each with a different exposure to p_t, where the assets consist of revenues from production, costs from operations, options to expand, and options to cease operations. In Appendix B we discuss the limitations of the single-factor assumption.

2.2. Basic (revenue-only) firm

Consider the cash flows associated with producing one unit of output good per unit time. The unit price of the good at date t, in real terms, is e^{p_t} and its log price is p_t. Assume the log price evolves as an Ornstein-Uhlenbeck process, that is,

$$\mathrm{d}p_t = (\mu - \theta p_t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t^p. \tag{5}$$

Here, $\mu_t = \mu - \theta p_t$. Mean reversion in real output prices is common in models with competition in the product market and can be thought of as follows. A firm that makes an innovation today can expect profitable cash flows in the near to medium term. In the long term, other firms will compete the advantage away; see, for example, Dixit (1989), Leahy (1993), or Novy-Marx (2006). Permanent real revenue growth in our model can only result through investment.

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We identify p_t with the single risk factor discussed in the previous subsection. The risk premium associated with the log of output price, $\mu_t - \mu_t^*$, is assumed to be constant and positive. Thus, the risk-adjusted cash flow at date t follows the same process as shown in Eq. (5) with a constant μ^* substituted for μ. When $\theta \to 0$, we let $\mu^* = r - \delta - \sigma^2/2$, where r is a constant real risk-free rate and $\delta > 0$ is a proportional net convenience yield. This simple setup allows us to observe the impact of costs, growth options, and shutdown options on return autocorrelation while maintaining analytic tractability. Since $\mu_t - \mu_t^*$ is constant, we can use Eqs. (4) to analyze return autocorrelation.

We assume that at any time, production may be permanently ended by a "disastrous" and idiosyncratic Poisson shock that is statistically independent of p_t. The arrival rate of such a shock is denoted as λ and is assumed to be constant. The present value of the revenue stream from this production, denoted V_t^B for "basic firm," is simply the value of a portfolio of forwards, each of which corresponds to unit production at some future date. Assuming the real interest rate is constant, this takes the form:

$$V^B(p_t) = \int_0^\infty e^{-(r+\lambda)\tau} \mathrm{E}_t^*[e^{p_{t+\tau}}]\,\mathrm{d}\tau = \int_0^\infty e^{-(r+\lambda)\tau} F(p_t, \tau)\,\mathrm{d}\tau, \tag{6}$$

where $\mathrm{E}_t^*[e^{p_{t+\tau}}]$ denotes the risk-adjusted expected value of $e^{p_{t+\tau}}$ conditional on date t information. Note that the effect of the Poisson shock is simply to increase the risk-adjusted rate of discounting cash flow from the real risk-free rate, r, to $r + \lambda$. The term $F(p_t, \tau)$ is the forward price for selling the output good at date $t + \tau$, and is given by

$$F(p_t, \tau) = \exp\left(e^{-\theta\tau} p_t + (1 - e^{-\theta\tau})\frac{\mu^*}{\theta} + \sigma^2 \frac{1 - e^{-2\theta\tau}}{4\theta}\right). \tag{7}$$

After a bit of manipulation, the firm value is calculated to be[6]

$$V^B(p_t) = \frac{1}{\theta}\int_0^1 s^{(r+\lambda)/\theta - 1} e^{p_t s + (1-s)\mu^*/\theta + \sigma^2 \frac{1-s^2}{4\theta}}\,\mathrm{d}s. \tag{8}$$

Letting $v_t^B \equiv \ln V_t^B$, in the case of the basic (revenue-only) firm we have

$$\beta^B(p_t) = \frac{\partial v_t^B}{\partial p_t}.$$

In Appendix A we establish the following:

Proposition 2. $\partial v_t^B/\partial p \in (0, 1]$ *and is one if and only if* $\theta = 0$; $\partial^2 v_t^B/\partial p^2 \geqslant 0$ *and the inequality is strict if and only if* $\theta > 0$.

To put Proposition 2 into words, the present value of our basic (revenue-only) firm exhibits positive expected returns and positive return autocorrelation. The expected returns of V_t^B are positive because the basic firm's value is a sum of forwards. The forward maturing at date $t + \tau$ has a sensitivity (factor beta) of $\partial \ln F(p_t, \tau)/\partial p_t > 0$. Each forward has a positive beta with respect to p_t and therefore each has a positive expected return. It follows that the basic firm is a portfolio long on such forwards, and that it has positive expected returns. The following lemma, while straightforward to prove, helps explain the source of positive return autocorrelation for our basic firm. The lemma is also very useful in explaining our subsequent results regarding return autocorrelation.

[6]Under our assumptions for $\theta \to 0$, $\lim_{\theta \to 0} V^B(p_t) = e^{p_t}/(\delta + \lambda)$.

Lemma 2.1. *Let* $v_1 \equiv \ln|V_1|$ *and* $v_2 \equiv \ln|V_2|$, *with* $V_1 + V_2 > 0$. *If both* v_1 *and* v_2 *are twice differentiable with respect to the factor* p, *and* $\omega = V_1/(V_1 + V_2)$, *then*

$$\frac{\partial^2 \ln(V_1 + V_2)}{\partial p^2} = \omega \frac{\partial^2 v_1}{\partial p^2} + (1-\omega)\frac{\partial^2 v_2}{\partial p^2} + \omega(1-\omega)\left(\frac{\partial v_1}{\partial p} - \frac{\partial v_2}{\partial p}\right)^2.$$

The key insight of the lemma is that the log convexity of a portfolio ($V_1 + V_2$) is not the portfolio of component log convexities. Even if V_1 and V_2 each have zero log convexity, combining the two leads to nonzero log convexity of $V_1 + V_2$ as long as their factor loadings on p are different. In the latter case, the cross-term is strictly positive if the portfolio is long both of its constituents—see Scheme 1 in Section 1.2. The cross-term is strictly negative if the portfolio is 'long-short' as in Scheme 2.

Lemma 2.1 explains why our basic firm (V_t^B) exhibits positive return autocorrelation. Since $\partial \ln F(p_t, \tau)/\partial p_t$ decreases with τ, longer-maturity forwards are less risky than near-term forwards. Notice that a single such forward has zero log convexity and therefore exhibits no return autocorrelation. However, Lemma 2.1 indicates that combining two such forwards with different maturities (i.e., different factor sensitivities) results in strictly positive log convexity. The positive log convexity of the resulting portfolio is maintained when additional forwards are added. If cash flows are not mean reverting (i.e., $\theta = 0$), then all forwards have the same riskiness, v_t^B is linear in p_t, and the log convexity is zero. It is important to understand that even in cases in which $\theta = 0$, firms with costs and real options may still exhibit either positive or negative return autocorrelation. Sections 2.3 and 2.4 explain these points.

2.3. Costs

We now add costs to the present value of revenues in Eq. (6), so that our model firm consists of different cash flow components, and consider the case of fixed production; we take up growth possibilities in Section 2.4. When production is fixed, there is no distinction between fixed and variable costs: Both types of costs reduce cash flows to equity holders. Likewise, interest expense also represents a reduction of cash flows to equity holders. Therefore, we assume that a constant and real total cost of K is incurred per unit time. The value of the firm with costs is now denoted V_t^C and is equal to the sum of risk-adjusted, discounted cash flows:

$$V^C(p_t) = \int_0^\infty \mathrm{E}_t^*[e^{p_{t+\tau}} - K] \cdot e^{-(r+\lambda)\tau}\,\mathrm{d}\tau, \tag{9}$$

where $\mathrm{E}_t^*[\cdot]$ denotes risk-adjusted expected value and $e^{p_{t+\tau}}$ denotes date $t + \tau$ revenue. As in Eq. (6), the real risk-free interest rate is assumed constant and the Poisson shocks increase the time discount rate by λ. Note that in the expression above, we assume that the firm has no limited liability option; we take up such options in Section 2.4. We also assume the firm pays the cost K until a Poisson shock arrives. Under these assumptions, the expression for V_t^C separates into the value of the basic firm, V_t^B from Eq. (8), minus the stream of discounted costs:

$$V^C(p_t) = V^B(p_t) - \frac{K}{r+\lambda}. \tag{10}$$

Thus, the above firm can be viewed as a portfolio that is long a risky asset with value $V^B(p_t)$ and short a risk-free asset with value $K/(r+\lambda)$. This situation resembles Scheme 2 in Section 1.1. The economic intuition there suggests that the addition of costs introduces negative autocorrelation in returns. This intuition is not precise, however, because it does not take into account the positive autocorrelation in the returns of V_t^B (see Proposition 2). To obtain a more precise characterization, we make use of Lemma 1 and set $V_1 = V_t^B$ and $V_2 = -K/(r+\lambda)$. Applying the lemma gives

$$\frac{\partial^2 \ln V_t^C}{\partial p_t^2} = \frac{V_t^B}{V_t^C}\left(\frac{\partial^2 v_t^B}{\partial p_t^2} - \frac{1}{V_t^C}\frac{K}{r+\lambda}\left(\frac{\partial v_t^B}{\partial p_t}\right)^2\right). \tag{11}$$

Keeping all other parameters constant, increasing K from zero will eventually lead to $\partial^2 \ln V_t^C/\partial p^2 < 0$ and thus negative conditional return autocorrelation. In particular, one can use Eq. (11) to solve for the maximum magnitude of costs consistent with positive return autocorrelation. As this argument suggests, the presence of costs can explain negative return autocorrelation. As is evident in Figs. 1 and 2, to account for significant positive return autocorrelation in firms calibrated to realistic data, one must incorporate growth options into our model.

2.3.1. *Costs and the Johnson (2002) model*

Like our revenue-only firm, the Johnson (2002) model of return autocorrelation has only positive cash flows (i.e., no costs). Does the inclusion of costs also significantly alter the level of return autocorrelation in the Johnson (2002) model? To answer this question, we add a constant interest rate perpetuity of costs, $(-K/r)$, to Johnson's model. Using the parameters from Table 1 of Johnson (2002), we calculate the minimum amount of net profit margin required to maintain a positive amount of return autocorrelation. Net profit margin is given by $(1 - K/D)$, where D is Johnson's current value of nonnegative cash flows to equity holders. Even in Johnson's most aggressive scenario (Scenario F of his Table 1), all of the positive return autocorrelation effects disappear if profit margins are below 76%. Moreover, the smallest profit margin required for positive return autocorrelation is no less than 73% when considering all scenarios. We conclude that adding realistic costs to Johnson's (2002) model can significantly alter and even reverse his results.

2.4. *Growth and shutdown options*

We now incorporate growth and limited liability options into our model firm. We are interested in answering the question: How do options affect a firm's return autocorrelation? Growth options are risky assets that increase in value more than assets in place when the firm performs well. One can therefore anticipate that growth options contribute positively to return autocorrelation as alluded to in Scheme 1 of Section 1.1. Limited liability options are akin to selling the firm's assets as the firm performs poorly. In bad times, such options limit the scope of financial distress and reduce risk as well as expected returns. Intuition therefore suggests that limited liability options also contribute positively to return autocorrelation.

We model both the firm's growth and limited liability options as perpetual (i.e., they never expire). This is particularly sensible for the limited liability option. With respect to the growth option, while competitive pressures may constrain a firm's ability to defer investment, in a price-taking environment such competitive effects are generally impounded into the underlying (e.g., mean-reversion in real output prices); thus save for exceptional circumstances, the first-order impact of growth options is well modeled by assuming they are also perpetual. Below we demonstrate that the impact of perpetual growth and limited liability options generally accords with the intuition described in the paragraph above.

Consider a firm that is currently producing one unit of output good per unit time and paying a fixed real cost of K, as in Section 2.3. Suppose, in addition, that the firm has an option to enhance its cash flow by investing I dollars, as well as a perpetual limited liability option. Finally, suppose that it is optimal for the firm to exercise the growth option if p_t, the log price of the output good, rises above $\bar{p}$, and it is optimal to exercise the limited liability option if p_t falls below $\underline{p}$. Under these assumptions, the value of the firm prior to the exercise of the options can be written as

$$V(p_t)^{\text{pre}} = V^C(p_t) + \alpha_+ \frac{U_+(p_t)}{U_+(\bar{p})} + \alpha_- \frac{U_-(p_t)}{U_-(\underline{p})}, \tag{12}$$

where $V^C(p_t)$ is the present value of revenues less costs in the absence of the options (see Eq. (10)), $\alpha_+(U_+(p_t))/(U_+(\bar{p}))$ is the value of the binary (cash-or-nothing) call option that never expires and pays α_+ dollars the first time p_t rises above $\bar{p}$, and $\alpha_-(U_-(p_t))/(U_-(\underline{p}))$ is the value of the binary put option that never expires and pays α_- dollars the first time p_t falls below $\underline{p}$. The call and put option payoffs, α_+ and α_-, solve the "value matching" conditions

$$\alpha_+ = V^{\text{post}}(\bar{p}) - \left(V^B(\bar{p}) - \frac{K}{r+\lambda} + \alpha_- \frac{U_-(\bar{p})}{U_-(\underline{p})} \right) - I, \quad \text{and} \tag{13}$$

$$\alpha_- = -\left(V^B(\underline{p}) - \frac{K}{r+\lambda} + \alpha_+ \frac{U_+(\underline{p})}{U_+(\bar{p})} \right). \tag{14}$$

Eq. (13) states that the payoff from exercising the growth option at $p_t = \bar{p}$ equals the value of the post-exercise firm, $V^{\text{post}}(\bar{p})$, less the value of current operations (the term in parentheses) and less the required investment, I. This guarantees that immediately after exercising the growth option and investing I, the firm's value is $V^{\text{post}}(\bar{p})$. Eq. (14) states that the payoff from exercising the limited liability option at $p_t = \underline{p}$ offsets the value of current operations (assumed negative at $\underline{p}$) so that the net value of the firm is zero immediately after exercise. The optimal values of $\bar{p}$ and $\underline{p}$ are determined by "smooth pasting" conditions that equate the marginal value of the pre-exercise firm to that of the post-exercise firm *at the exercise point*; see Dumas (1991).

Each of the perpetual \$1 binary options, $(U_+(p_t))/(U_+(\bar{p}))$ and $(U_-(p_t))/(U_-(\underline{p}))$, must satisfy the time-independent differential equation associated with the risk-adjusted dynamics of the underlying. In the case of Ornstein-Uhlenbeck dynamics combined with the Poisson shock, this equation is

$$\frac{\sigma^2}{2} U_{pp} + (\mu^* - \theta p) U_p - (r+\lambda) U = 0. \tag{15}$$

The impact of the Poisson event on the value of these options is accounted for by shifting the discount rate from r to $r+\lambda$. This ordinary differential equation has two fundamental solutions, one increasing and the other decreasing in p_t. The first is associated with $U_+(p)$, while the second is associated with $U_-(p)$:

$$U_\pm(p) = H\left(-\frac{r+\lambda}{\theta}, \pm\frac{((\mu^*/\theta)-p)\sqrt{\theta}}{\sigma}\right), \tag{16}$$

where $H(v,z)$ is a generalized Hermite function of order v.

Eqs. (12)–(16) formally model the firm value as a portfolio of revenues, costs, a growth option, and a limited liability option. The following proposition pins down the impact of the options on the return autocorrelation of our model firm.

Proposition 3. *Suppose $V^C(p_t)$ in Eq.* (10) *is positive and exhibits negative conditional return autocorrelation at p_t. Then the conditional return autocorrelation of $V^{\text{pre}}(p_t)$ in Eq.* (12) *is strictly greater than the conditional return autocorrelation of $V^C(p_t)$.*

At the end of Section 2.3 we note that under realistic parameters, $V^C(p_t)$ exhibits negative return autocorrelation. Proposition 3 says that under realistic parameters, adding growth or limited liability options to the firm's assets strictly increases return autocorrelation. This confirms the intuitive analysis given earlier in this section and in the Introduction. One can also demonstrate that as the magnitude of costs relative to the other assets in the firm decreases, return autocorrelation eventually turns positive.

Example. To understand the economic significance of Proposition 3, we show that over the span of one year the instantaneous annualized expected return from holding $U_+(p_t)$, the perpetual call in Eq. (16), increases dramatically with p_t. Assume the annual real interest rate is 1%, the mean reversion half-life is 18 months, the annual revenue growth volatility is 0.30, the annual Sharpe Ratio for revenue risk is 0.35, and $\lambda = 1\%$. Then, as the log revenue p_t shifts from one unconditional standard deviation below its mean to one unconditional standard deviation above its mean, the expected return on the perpetual call option increases from 2.25% to 17.3%. This represents a substantial increase in expected returns and demonstrates positive return autocorrelation.

The analysis in this section thus far suggests that, ceteris paribus, firms with more valuable growth options and low costs have higher return autocorrelation and may serve as a basis for enhanced momentum strategies. The difficulty in verifying such a conjecture is that the parameters, $\bar{p}, \underline{p}$, and $\alpha_\pm$ in Eq. (12) are endogenously determined and may themselves depend on $\bar{K}$. To ascertain whether a reasonably parameterized firm exhibits positive return autocorrelation, and whether enhanced momentum strategies are feasible, one must resort to a numerical investigation. We do this in Section 3.

2.4.1. *Options with finite expiry*

Our model considers only perpetual options. It is valuable to have some indication of whether and how options with finite time to expiry might impact our conclusions regarding return autocorrelation. Readers who are more interested in the calibration of the model with perpetual options can skip ahead to Section 3.

Proposition 3 relies on the fact that the perpetual call and put options have nonnegative log convexity. This is not generally true for other types of options. For instance, due to its implicit leverage, the Black-Scholes European call option has negative log convexity (see the example of Section 2.1). By contrast, any implicit leverage is completely discounted for an American-style option that does not expire. Does a portfolio containing European-style options and the underlying exhibit positively autocorrelated returns? After addressing this question we extend our analysis to finite-maturity American-style options. The case of perpetual options covered in Proposition 3 corresponds to increasing the maturity of the American-style options to infinity.

European-style options: Consider a portfolio that has weight $\omega_{\text{Call}} \in (0,1)$ in Black-Scholes options of fixed strike price and maturity, and weight $1-\omega_{\text{Call}}$ in the underlying stock. Because the options are riskier than the underlying stock and the portfolio is long both assets, the options' weight in the portfolio increases with the stock price. One might expect the portfolio to become more risky as the stock price rises, and thus to exhibit positive return autocorrelation (the effect described by Scheme 1 of Section 1.1). On the other hand, due to their implicit leverage, the options become less risky as the stock price increases. As we now show, it is not possible to unambiguously sign the return autocorrelation of this portfolio. To see this, let the stock and option components play the roles of V_1 and V_2, respectively, in Lemma 2.1. The log price of the stock plays the role of p. Let $\ln(C^E)$ be the natural log of the call option value. Lemma 1 can now be used to establish that the portfolio return autocorrelation is positive if and only if the weight of the options in the portfolio satisfies

$$\omega_{\text{Call}} < 1 + \frac{(\partial^2/\partial p^2)\ln(C^E)}{(1-(\partial/\partial p)\ln(C^E))^2}. \tag{17}$$

Fig. 3 illustrates the case in which a portfolio containing shares of stock and call options (written on the stock) exhibits positive return autocorrelation. When the stock is trading at the option's strike price the moneyness is equal to one. If 52% or less of the portfolio value consists of call options, the portfolio will exhibit positive return autocorrelation when the moneyness is one. The graph also shows that as the moneyness increases (and risk decreases), the maximum portfolio weight of call options consistent with positive log convexity decreases. Although not shown in the graph, as the volatility of the stock increases or the maturity of the call option increases, the maximum weight (ω_{Call}) in options also decreases. To summarize: the impact of European call options on a portfolio's return autocorrelation is generally ambiguous.

The case of European puts is unambiguous because these options have negative log convexity. One can show that a positive risk premium portfolio containing stocks and put options always exhibits positive convexity and therefore positive return autocorrelation. The intuition is straightforward: the associated replicating portfolio will always be long stock and cash; as the stock price decreases, the replicating portfolio sells the underlying and thereby reduces risk.

American-style options: We now argue that an American-style option can be viewed as a combination of a European option and a binary option with time-varying payoffs. The portfolio implication is that any option with a finite expiry will combine properties of the perpetual options considered in Proposition 3 with the properties of the European options considered above.

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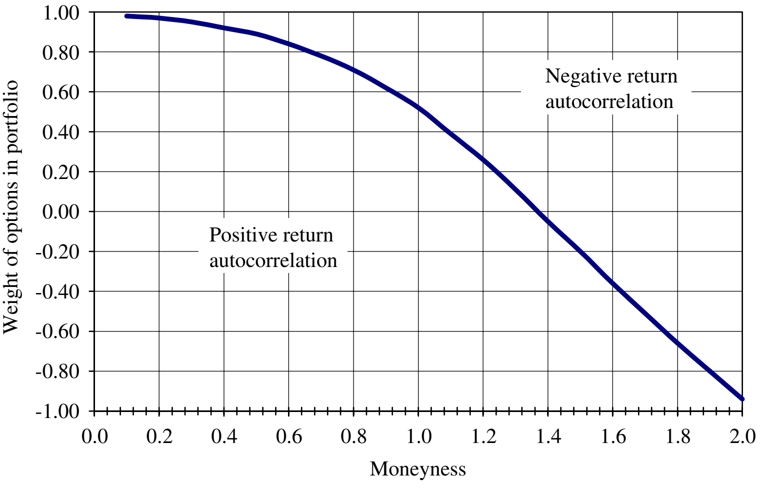

Fig. 3. Return autocorrelation of a portfolio containing shares of stock and European call options. The figure shows when a portfolio exhibits positive return autocorrelation and when it exhibits negative return autocorrelation. The x-axis is labeled "moneyness" and represents the ratio of the current stock price to the options' strike price. The y-axis shows the weight of the call options in the portfolio. When the stock price equals the strike price, the moneyness is one, and a portfolio with a weight of 0.52 or less in call options exhibits positive return autocorrelation. If call options have a weight over 0.52, the portfolio exhibits negative return autocorrelation. The strike price of the call option is \$100, the annual volatility is 30%, the risk free rate is 6.0%, the dividend rate is 4.00%, and the time to maturity is one year.

Baron-Adesi and Whaley (1987) approximate an American call or put with time τ left to maturity as

$$C_t^A \approx C_t^E + \alpha_+(\tau)\frac{U_+(p_t)}{U_+(\bar{p}(\tau))} \quad \text{and} \quad P_t^A \approx P_t^E + \alpha_-(\tau)\frac{U_-(p_t)}{U_-(\underline{p}(\tau))}.$$

In the expressions above, C_t^E is the value of a corresponding European call option. The term containing $(U_+(p_t))/(U_+(\bar{p}(\tau)))$ is the value of the timing option inherent in the American call. The timing option is proportional to the value of a perpetual binary call option. The difference is that the exercise boundary $\bar{p}(\tau)$ and option payoff $\alpha_+(\tau)$ depend on the time left to maturity.[7] A similar statement applies to the put option. For the class of processes we are considering, $C_t^E \to 0$ and $P_t^E \to 0$ as $\tau \to \infty$. Intuitively, this says that the present value of receiving one unit of the commodity in the future vanishes with the horizon. On the other hand, $\alpha_\pm(\tau)$ stays finite even as $\tau \to \infty$. Thus, all the value of the American call/put option resides in the timing component of the option as the maturity grows. As stated above, American options with finite expiry combine properties of the perpetual options with the properties of European options.

What is the impact on return autocorrelation when one combines an American option with its underlying to form a portfolio? While portfolios containing European call options

[7]While Baron-Adesi and Whaley (1987) establish this for an underlying that follows a geometric Brownian motion (GBM), their approach is easily extended to the family of Ornstein-Uhlenbeck processes (which nests GBM).

may exhibit negative return autocorrelation as a result of their inherent leverage (Fig. 3), American options with long maturities discount the leverage to zero. An analysis similar to that in the proof of Proposition 3 establishes that a portfolio long stocks and American call options written on the stock eventually exhibits positive return autocorrelation as the maturity of the option increases. Likewise, a positive risk premium portfolio long stocks and American put options will always have positive log convexity. The return autocorrelation of a portfolio long the underlying and long an option is summarized below[8]

Option	log convexity	Contribution to return autocorrelation
European call	<0	Ambiguous
European put	<0	Positive
American perpetual (timing) call	$\geqslant 0$	Positive
American perpetual (timing) put	$\geqslant 0$	Positive
American call	Ambiguous	Ambiguous
American put	Ambiguous	Positive

Overall, it should be clear that the interactions among revenues, costs, and options in the firm's portfolio can lead to complicated dynamics in the firm's conditional return autocorrelation. Except in limiting cases, one cannot analytically sign or determine the magnitude of the resulting effects. We therefore turn to numerical calibration.

3. Numerical analysis

We numerically analyze a model firm based on Eqs. (12)–(16) in order to quantify the trade-off between costs (negative autocorrelation) and growth options (positive autocorrelation). We begin by selecting benchmark parameters based on market data. We then perform a sensitivity analysis of the return autocorrelation exhibited by the benchmark firm. We end with a study of momentum strategies by considering a population of model firms. Our goal is to demonstrate that our simple model is capable of capturing the cross-sectional properties of returns found in previous empirical studies. We also want to numerically test for "enhanced" momentum strategies. In Section 4, we compare the numerical results with our empirical results using CRSP/Compustat data.

3.1. The model firms

Each of our model firms starts its life with a one-time irreversible growth option.[9] All cash flows are real (i.e., adjusted for inflation). The per-unit log price of the firm's output at

[8] The portfolio is assumed to have a positive beta.

[9] While the one-time availability of a positive net present value (NPV) investment opportunity may seem overly simplistic, it allows us to capture the existence of growth options without the additional burden associated with an infinite set of nested options. We do not preclude the possibility that the firm may grow by reinvesting its cash flows in zero NPV projects instead of paying dividends to share holders. This does not change the risk-return prospects of the firm. The solution to the case with an infinite number of nested growth options is similar in form

date t is p_t. Equity holders are entitled to $(e^{p_t} - K^{\text{pre}})$ per unit time (i.e., the cash flows of this firm). When a firm exercises its growth option, it pays an expansion cost of I and grows in physical size. In particular, the firm goes from producing one unit of output good per unit of time to $\xi > 1$ units of output good per unit of time, and cash flows to equity holders grow to $\xi(e^{p_t} - K^{\text{post}})$. Expansion also changes both the costs, from K^{pre} to K^{post}, and the probability per unit time the firm experiences a Poisson shock (leading to immediate bankruptcy), from λ^{pre} to λ^{post}. After exercising the growth option, the firm continues to retain its limited liability option.

The equity market values for pre-exercise and post-exercise firms are given below and come directly from Eq. (12). The α_+ term corresponds to the growth option of the pre-exercise firm while the α_- terms correspond to the limited liability options. The value of the revenue-only firm (V_t^B) is given in Eq. (8).

$$V_t^{\text{pre}} = V_t^B(p_t, \lambda^{\text{pre}}) - \frac{K^{\text{pre}}}{r + \lambda^{\text{pre}}} + \alpha_+^{\text{pre}} \frac{U_+(p_t, \lambda^{\text{pre}})}{U_+(\bar{p}, \lambda^{\text{pre}})} + \alpha_-^{\text{pre}} \frac{U_-(p_t, \lambda^{\text{pre}})}{U_-(\underline{p}^{\text{pre}}, \lambda^{\text{pre}})}, \quad \text{and}$$

$$V_t^{\text{post}} = \xi\left(V_t^B(p_t, \lambda^{\text{post}}) - \frac{K^{\text{post}}}{r + \lambda^{\text{post}}} \right) + \alpha_-^{\text{post}} \frac{U_-(p_t, \lambda^{\text{post}})}{U_-(\underline{p}^{\text{post}}, \lambda^{\text{post}})}.$$

The pre-exercise firm exercises its growth option at $p_t = \bar{p}$. The pre- and post-exercise firms exercise their limited liability options at $p_t = \underline{p}^{\text{pre}}$ and $\underline{p}^{\text{post}}$, respectively. As discussed earlier, the firm's value must satisfy the boundary (value matching and smooth pasting) conditions

$$V_t^{\text{pre}}(\underline{p}^{\text{pre}}, \lambda^{\text{pre}}) = 0 \quad \text{and} \quad \left.\frac{\mathrm{d}V_t^{\text{pre}}}{\mathrm{d}p}\right|_{\underline{p}^{\text{pre}}} = 0, \tag{18}$$

$$V_t^{\text{post}}(\underline{p}^{\text{post}}, \lambda^{\text{post}}) = 0 \quad \text{and} \quad \left.\frac{\mathrm{d}V_t^{\text{post}}}{\mathrm{d}p}\right|_{\underline{p}^{\text{post}}} = 0, \tag{19}$$

$$V_t^{\text{pre}}(\bar{p}, \lambda^{\text{pre}}) + I = V_t^{\text{post}}(\bar{p}) \quad \text{and} \quad \left.\frac{\mathrm{d}V_t^{\text{pre}}}{\mathrm{d}p}\right|_{\bar{p}} = \left.\frac{\mathrm{d}V_t^{\text{post}}}{\mathrm{d}p}\right|_{\bar{p}}. \tag{20}$$

The value matching condition at the point of expansion—shown in the third set of equations—requires that the post-exercise firm's value equal that of the pre-exercise firm *plus* an investment "strike" price. We assume investment capital is raised through the issuance of additional equity.

Benchmark parameter values: Our model firm has 17 parameters including α_+^{pre}, α_-^{pre}, α_-^{post}, $\underline{p}^{\text{pre}}$, $\underline{p}^{\text{post}}$, and $\bar{p}$. Eqs. (18)–(20) pin down six of these parameters. We normalize the unit price of a good by setting $\mu = -\sigma^2/4$. This is equivalent to setting the long-run expected revenues from a unit of production to one.[10] Historical data can be used to estimate seven additional parameters. We are left with three undetermined or "free" parameters that can be used to target cross-sectional return properties. Note that one can write the risk premium as $\mu - \mu^* = \rho \; SR \; \sigma$, where SR is the maximal Sharpe Ratio

(*footnote continued*)
to our single-option solution. The difference is that the infinite case features an infinite set of optimal investment thresholds and an infinite set of option coefficients—the α_+'s and α_-'s in Eqs. (13) and (14).

[10]The expected revenues are given by the expression for $F(p_t, \tau)$ with μ substituted for μ^* (i.e., the unadjusted forward price). This expression approaches one for $\tau \to \infty$ if $\mu = -\sigma^2/4$.

attainable in the economy and $-\rho$ is the instantaneous correlation of p_t with the pricing kernel (see Duffie and Zame, 1989). Given an estimate of SR, one should view ρ as a model parameter. Appendix C fully describes our estimation approach and Table 1 reports the resulting benchmark parameters.

Table 1, Panel A reports parameter estimates. The term $Pr^{\text{pre}}(exercise)$ is the annualized probability that a pre-exercise firm becomes a post-exercise firm by exercising its growth option; its value determines $\bar{p}$. The terms $Pr^{\text{pre}}(exit)$ and $Pr^{\text{post}}(exit)$ are the annualized *total* rates of exit for pre-and post-exercise firms, respectively, where each is the sum of the corresponding Poisson shock probability (λ^{pre} or λ^{post}) and rate of exit implied by the endogenous default thresholds ($\underline{p}^{\text{pre}}$ or $\underline{p}^{\text{post}}$). Since the values of $Pr^{\text{pre}}(exit)$ and $Pr^{\text{post}}(exit)$ are set to match the observed exit rates of firms in data, the values of λ^{pre} and λ^{post} can be deduced from these estimates and knowledge of $\underline{p}^{\text{pre}}$ and $\underline{p}^{\text{post}}$. We set SR to 0.5, consistent with other studies (see Campbell, 2003). Finally, we set the real risk-free rate, r, to 2.0%, consistent with the historic averages of three-month T-bill rates deflated by the Producer Price Index between 1963 and 2004, as reported by the St. Louis Federal Reserve Bank.[11]

Table 1, Panel B reports benchmark values for the free parameters. We vary these parameters to match the market, size, and momentum return premia when we consider a heterogeneous population of firms in Section 3.4. The parameter q, discussed in Section 3.3, is used to calibrate book values of pre-exercise firms so as to match the value premium. We discuss the economic significance of the benchmark values in Section 3.4 as well. Panel C lists the endogenously determined parameters. Their values are calculated from Eqs. (18)–(20).

The large value of ξ we infer from the data implies that much of the pre-exercise firm's value arises from its expansion option. This option value is generally sufficiently valuable that it would be suboptimal for a pre-exercise firm to exercise its limited liability option except in an extremely unlikely contingency or if its costs are implausibly high. Thus, for many firm parameterizations, $\underline{\alpha}^{\text{pre}}$ is essentially zero, and the observed exit rate, $P^{\text{pre}}(exit)$, of 1.1% can only be obtained in the model by setting $\lambda^{\text{pre}} = 0.011$. Essentially, pre-exercise firms only exit due to Poisson shocks. This observation also justifies the use of Poisson shocks in our model as they are required to fit the observed exit rates of pre-exercise firms.

3.2. Benchmark firms

To help visualize return autocorrelation in our model firms, we choose two parameterizations to illustrate key points. Fig. 1, Panel A graphs the log value of a high cost firm. Panel B graphs the factor sensitivities or betas. The instantaneous excess expected return of a firm is given by its beta times $\mu - \mu^*$ (see Eq. (1)). The parameters for the high cost firm are all given in Table 1 except K is now set to 0.775. Positive return autocorrelation is evident when the log value of the firm is convex—an increase in the value of the pre-exercise firm is synonymous with an increase in the firm's beta (i.e., expected returns). An increasing beta obtains in Panel B for the pre-exercise firms only. Fig. 1 also shows that high costs and the absence of a growth option cause the log value to

[11]We use the Producer Price Index rather than the Consumer Price Index because we are deflating producer revenues and costs.

Table 1
Parameter values

The table lists parameter values used in the numerical analysis. Panel A shows parameters calculated from market data. Appendix C give details on estimating the parameter values. Panel B shows the free parameters. We adjust these parameters in order to match specific moments from market data. Panel C shows parameters that are endogenously determined in our model.

Panel A: Parameters matched to actual market data

Parameter	Description	Base case	Additional notes
σ	Volatility of log revenue process	30%	Estimated as the annual volatility of revenue growth. μ is set to $-\sigma^2/4$
$\Pr^{pre}(exercise)$	Probability of pre-exercise firm becoming a post-exercise firm	0.030	This annual probability determines $\bar{p}^{pre}$
$\Pr^{pre}(exit)$	The probability of exit (default) for pre-exercise firm	0.011	This annual probability equals the Poisson arrival rate (λ^{pre}) plus probability of endogenous shutdown (from $\underline{p}^{pre}$)
$\Pr^{post}(exit)$	The probability of exit (default) for post-exercise firm	0.0035	This annual probability equals the Poisson arrival rate (λ^{post}) plus probability of endogenous shutdown (from $\underline{p}^{post}$)
ξ	Magnitude of growth option	30	See Appendix C for details relating to the estimation of ξ.
SR	Sharpe Ratio	0.50	Maximal possible Sharpe Ratio in the economy. $\mu-\mu^*$ is set to $\rho SR\sigma$
r	Real risk-free rate	2%	Based on historic averages of three month Tbills and the Producer Price Index from St. Louis Federal Reserve Bank (1963–2004)

Panel B: Free parameters

Parameter	Description	Base case
K	Costs	0.650
θ	Rate of mean reversion of log revenue process	0.455
ρ	Magnitude of correlation of p_t with the pricing kernel	0.700
q	Ratio of market-to-book of pre- versus post-expansion firms at the point of expansion	1.260

Panel C: Endogenously determined parameters

Parameter	Description
$\underline{p}^{pre}$	Shutdown price for pre-exercise firms
$\underline{p}^{post}$	Shutdown price for post-exercise firms
α_+^{pre}	Growth option payoff for pre-exercise firms
α_-^{pre}	Shutdown option payoff for pre-exercise firms
α_-^{post}	Shutdown option payoff for post-exercise firms
I	Cost paid by a pre-exercise firm in order to exercise its growth option

be *concave* for the post-exercise firm, thus exhibiting negative return autocorrelation. Again, Panel B provides insights: as the post-exercise firm value rises, its beta decreases leading to lower expected returns.

Fig. 2 examines pre-exercise and post-exercise versions of our benchmark firm with low costs ($K = 0.525$). Costs are sufficiently low that a post-exercise firm acts like a low- and constant-risk asset (i.e., beta is low and constant). The pre-exercise firm is affected by the growth option, and its log value is convex particularly near the exercise threshold $\overline{p}$. It is also near $\overline{p}$ that the pre-exercise firm exhibits the highest return autocorrelation.

Fig. 1 clearly establishes that the presence of valuable growth options can reverse the log concavity caused by costs. Our pre-exercise firm exhibits positive return autocorrelation over a wide range of p_t despite the presence of high costs.

3.3. Sensitivity of return autocorrelation

We investigate the sensitivity of return autocorrelation to the underlying parameters in our model. One way to quantify the amount of return autocorrelation a given firm exhibits is to compare the expected return of the firm when the firm's value is high (e.g., after a large, positive shock) to the expected return when firm value is low (e.g., after a large, negative shock):

$$RetAutoCorr = \sigma\rho SR\left(\frac{\beta^{\text{pre}}(p_h) + \beta^{\text{post}}(p_h)}{2} - \frac{\beta^{\text{pre}}(p_l) + \beta^{\text{post}}(p_l)}{2}\right). \tag{21}$$

We set p_h, the "high value" point of the firm, to one unconditional standard deviation above the median of the price process and we set p_l, the "low value" point of the firm, to one unconditional standard deviation below the median. Since the same firm can exist in two forms (pre-exercise or post-exercise), Eq. (21) averages over the pre- and post-exercise firm betas at p_h and p_l, respectively. Table 2 shows the sensitivity of *RetAutoCorr* to changes in the model parameters. Panel A, for instance, shows that as volatility increases from 10% to 50% return autocorrelation increases by $1.76\% = 0.044 \times (0.50 - 0.10)$. This is largely due to the increased dispersion implied by increasing σ; a similar effect can be seen when SR or ρ is increased. Increasing ξ also results in higher return autocorrelation, consistent with the higher value of the growth option. Increasing $Pr^{\text{pre}}(exercise)$ from 0.020 to 0.040 increases return autocorrelation by $1.01\% = 0.506 \times (0.04 - 0.02)$. Increasing $Pr^{\text{post}}(exit)$, on the other hand, decreases the value of the post-exercise firms and subsequently lowers the option value for the pre-exercise firms. Interestingly, increasing $Pr^{\text{pre}}(exit)$ increases return autocorrelation with a sensitivity similar in magnitude and sign as the sensitivity to the risk-free rate. The reason is that the changes in $Pr^{\text{pre}}(exit)$ are almost exclusively absorbed by changes in λ^{pre}. This, in turn, affects $r + \lambda^{\text{pre}}$, the risk-adjusted discount rate of a pre-exercise firm's cash flows—much as a change in r would. A higher interest rate increases return autocorrelation by increasing the weight of the perpetual option in the pre-exercise firm portfolio at the expense of the levered component.

The sensitivity of *RetAutoCorr* is highly nonlinear with respect to σ, θ, and K. In calculating these sensitivities we choose values to convey a broad rather than local sensitivity to the parameters. For example, Table 2, Panel B reports a sensitivity with respect to costs (K) of -0.117. This result comes from measuring sensitivity over a cost range of 0.525 to 0.775, and is consistent with the intuition from Section 2.3. When we measure the sensitivity of *RetAutoCorr* at small intervals around $K = 0.775$ we find it is

Table 2
Return autocorrelation sensitivity

The table reports the sensitivity of return autocorrelation to parameters in our model firm. Return autocorrelation is abbreviated as "*RetAutoCorr*" and is defined as a firm's expected returns when revenues are high versus low relative to the median. Panel A reports the sensitivity of return autocorrelation to model parameters (matched parameters). Panel B reports the sensitivity of return autocorrelation to model parameters (free parameters). Panel C estimates the sensitivity of return autocorrelation to the market-to-book ratio.

Panel A: Sensitivity of return autocorrelation to model parameters (matched parameters)

Parameter	Description	Parameter low value	Parameter high value	Return autocorrelation sensitivity
σ	Volatility of log revenue process	10%	50%	0.044
$Pr^{pre}(exercise)$	Probability of pre-exercise firm becoming a post-exercise firm	0.020	0.040	0.506
$Pr^{pre}(exit)$	The probability of exit (default) for pre-exercise firm	0.006	0.016	0.440
$Pr^{post}(exit)$	The probability of exit (default) for post-exercise firm	0.002	0.005	−0.035
ξ	Magnitude of growth option	20	40	0.0005
SR	Sharpe Ratio	0.350	0.650	0.058
r	Risk-free rate (real)	0.015	0.025	0.406

Panel B: Sensitivity of return autocorrelation to model parameters (free parameters)

Parameter	Description	Parameter low value	Parameter high value	Return autocorrelation sensitivity
K	Costs	0.525	0.775	−0.117
θ	Rate of mean reversion of log revenue process	0.380	0.530	−0.051
ρ	Correlation of p_t with the pricing kernel	0.600	0.800	0.043

Table 2 (*continued*)

Panel C: Sensitivity of return autocorrelation to the market-to-book ratio

We create 2,187 different firm parameterizations by varying the seven parameters shown at the bottom of the page. For each of the 2,187 different "firms," we calculate its return autocorrelation measure (*RetAutoCorr*) and its median market-to-book ratio. We then regress the return autocorrelation measure on a constant and the market-to-book ratio and present results below.

$$RetAutoCorr = \gamma_0 + \gamma_1 \left(\frac{M}{B}\right) + \varepsilon$$

	γ_0	γ_1	Adj R^2
Coef	−0.0268	0.0645	0.318
(*t-stat*)	*(−21.14)*	*(31.97)*	

Parameter	Description	Parameter low value	Parameter base case	Parameter high value
$Pr^{pre}(exercise)$	Probability of pre-exercise firm becoming a post-exercise firm	0.0200	0.0300	0.0400
$Pr^{pre}(exit)$	The probability of exit (default) for pre-exercise firm	0.0060	0.0110	0.0160
$Pr^{post}(exit)$	The probability of exit (default) for post-exercise firm	0.0020	0.0035	0.0050
ξ	Magnitude of growth option	20	30	40
K	Costs	0.525	0.650	0.775[a]
θ	Rate of mean reversion of log revenue process	0.380	0.455	0.530
ρ	Correlation of p_t with the pricing kernel	0.60	0.70	0.80

[a]We use $\min\{0.775, K^{max}\}$ where K^{max} is the highest cost consistent with observed exit rates $Pr^{pre}(exit)$ and $Pr^{post}(exit)$.

-0.75, while it is near zero for small intervals around $K = 0.65$. The nonlinearity can be traced to the dependence of the endogenous shutdown threshold on σ, θ, and K.

Table 2, Panel C shows the sensitivity of return autocorrelation to changes in the market-to-book ratio. Because book value is not an underlying parameter of our model, we must calculate a proxy for it. The book value of assets acquired by exercising the growth option is I; the book value of pre-exercise assets, B^{pre}, is not known. Our proxy variables are as follows:

$$\left(\frac{M}{B}\right)^{\text{pre}}(p) = \frac{V^{\text{pre}}(p)}{B^{\text{pre}}} \quad \text{and} \quad \left(\frac{M}{B}\right)^{\text{post}}(p) = \frac{V^{\text{post}}(p)}{B^{\text{pre}} + I}. \tag{22}$$

We set

$$\left(\frac{M}{B}\right)^{\text{pre}}(\bar{p}) = q \times \left(\frac{M}{B}\right)^{\text{post}}(\bar{p}). \tag{23}$$

Here, $q > 1$ reflects the greater weight of intangible assets in a pre-exercise firm just prior to exercising its growth option. If q is known, we can solve for B^{pre}. We choose $q = 1.26$ to match the value premium in our cross-sectional calibration in Section 3.4.

We calculate different market-to-book ratios for $3^7 = 2,187$ different firm parameterizations. The firms are generated by varying seven parameters over a low value, base case value, and a high value. The parameter values are shown in Table 2, Panel C.[12] To calculate the overall sensitivity of $RetAutoCorr$ to the market-to-book ratio, we run the cross-sectional regression

$$RetAutoCorr = \gamma_0 + \gamma_1 \left(\frac{M}{B}\right) + \varepsilon,$$

where $(\frac{M}{B}) = \frac{1}{2}((\frac{M}{B})^{\text{pre}}(p_m) + (\frac{M}{B})^{\text{post}}(p_m))$ and p_m is the median log price of the firm. Table 2, Panel C shows that return autocorrelation increases as the market-to-book ratio increases. The regression coefficient is 6.45% and the R^2 is high. About half of the explained variation is due to variation in costs. The remainder corresponds to variation in the other, not readily observable, six variables listed in Panel C of Table 2. The implication is that the market-to-book ratio is a good proxy for unobservable variables that determine high conditional autocorrelation in returns.

3.4. Numerical analysis of momentum strategies

We end our numerical analysis by testing whether different parameterizations of our model can generate momentum strategy profits as in Jegadeesh and Titman (1993). At the same time we attempt to roughly match the target cross-sectional return moments. A secondary goal of our analysis is to examine insights gleaned from the sensitivities calculated in Section 3.3. These sensitivities show that return autocorrelation is high in firms with high σ, low costs, and high market-to-book ratios. Can we generate *enhanced momentum strategies*, that is, strategies that produce higher momentum profits than those in Jegadeesh and Titman (1993), based on observable firm attributes?

[12] Some of the combinations involving high costs ($K = 0.775$) result in default rates higher than those fixed by $Pr^{\text{post}}(exit)$. In such a case, we set the "high cost" value for K to equal the highest cost figure that is consistent with the assumed value of $Pr^{\text{post}}(exit)$.

We construct a population of 840 representative firms that is heterogeneous in revenue volatility, costs, market value, market-to-book, and the past one-quarter's returns. Each firm is characterized by: (i) one of seven firm parameters; (ii) an indicator corresponding to whether the firm is pre- or post-exercise; (iii) a value for the firm's current log revenue; (iv) the last one-quarter's historic returns, which comprise a market and idiosyncratic component; and (v) a frequency corresponding to the relative number (not value) of each representative firm in the population. From this information we can also calculate the expected return for each firm. We can now analyze the expected returns of portfolios sorted on past returns as well as various other firm attributes—for example, size and book-to-market.

The seven different sets of parameters are chosen around the benchmark values shown in Table 1. We refrain from varying more parameters than we must for the sake of parsimony. Only σ and K are varied so that we can construct enhanced momentum strategies based on volatility and costs.[13] The ranges of σ and K are shown in Table 3, Panel A. We cannot have high costs ($K = 0.775$) along with high volatility ($\sigma = 0.50$) while keeping the exit rates of pre-exercise firms in line with market data. In order to keep the average parameters close to the base-case firm we exclude the low cost/low volatility ($K = 0.525/\sigma = 0.10$) case as well. The initial log revenues and past one-quarter's returns are consistent with a simple assumption over the steady state distribution of firms.[14] Every pre-exercise firm, regardless of its parameters, receives a frequency weight of 0.90 while every post-firm receives a frequency weight of 0.10. These frequencies are consistent with the number of pre-exercise and post-exercise firms estimated from the data and detailed in Appendix C. Appendix D provides full details on how the population of 840 firms is generated.

Matching target moments: We vary our free parameters θ, ρ, and the base-case K. We also vary q from Eq. (23). Our goal is to target the market, size, value, and momentum premia. The first three target values are based on monthly decile-sorted data from Ken French's web site. The momentum premium is based on CRSP/Compustat data (see Table 7, Panel A).

Table 3, Panel B shows that our "numerical" firms produce a quarterly population risk premium of 0.98%, a size premium of 2.12%, a value premium of 2.23%, and a momentum premium of 1.64%—all of which are within statistical tolerance of their target values. When sorting over firm attributes, we take account of the different frequencies assigned to the different firms. The book-to-market and momentum (past returns) portfolios are long the top decile of firms and short the bottom decile. Size portfolios are long the bottom decile and short the top decile. Our portfolio returns are all value-weighted.

How reasonable are the free parameter values needed to match target moments? Benchmark costs of $K = 0.65$ initially appear a little low. Given our choice of $-\mu = \sigma^2/4$ and the benchmark values of $\theta = 0.455$ and $\sigma = 0.30$, the unconditional mean of costs over

[13]In order to simultaneously match the value premium and momentum premium, we need the presence of both high cost and low cost firms.

[14]Our methodology effectively discretizes the steady state distribution of firms on a grid. For the steady state distribution, we use the unconditional probability distribution of the log revenue process (see Appendix D for details). A more orthodox methodology might randomly draw 10,000 initial log revenue values, assume some process for firm entry, and simulate returns until a steady state is reached. By assuming the steady state distribution, our procedure is less numerically intensive and not prone to simulation error.

Table 3
Numerical analysis of momentum strategies

The table reports results of different momentum strategies. Panel A shows the two parameters we vary: revenue volatility (σ) and costs (K). Panel B shows target moments and values obtained in our population of firms. Panel C shows results of enhanced momentum strategies. Base-case parameter values are given in Table I, Panel A.

Panel A: Parameters considered

Cost (K) values	Revenue volatility (σ) considered		
	0.10	0.30	0.50
0.525	No	Yes	Yes
0.650	Yes	Yes	Yes
0.775	Yes	Yes	No

Panel B: Target moments (quarterly)

	Target values (%)	Population moments from model firms (%)
Market risk premium	1.55	0.98
Size premium	1.24	2.12
Value premium	1.79	2.23
Momentum premium	2.66	1.64

Panel C: Enhanced momentum strategy returns (quarterly)

Momentum strategy #1 (%)		Momentum strategy #2 (%)		Diff. in strategies (%)
High σ Firms	1.92	Low σ Firms	0.94	0.98
Low Cost Firms	3.27	High Cost Firms	0.62	2.65
High M/B Firms	3.49	Low M/B Firms	−0.29	3.78
Up Markets	3.09	Down Markets	0.20	2.89

revenues in our model is 0.72. The median profit margin for firms in our data is close to 4.5% when profit margin is measured as income before extraordinary items divided by sales. Profit before extraordinary items includes taxes while our model firms do not. Also, non-cash expenses such as depreciation are omitted from our model but play a role in data. The average of costs of goods sold (CGS) over sales is 72% in market data and the average of CGS over total assets is 75%. These numbers are much closer to our benchmark values.

A mean reversion parameter value of 0.455 implies that a positive cash flow shock has a half-life of 1.5 years or 18 months. This value seems reasonable when one considers that momentum profits persist for about one year (Jegadeesh and Titman, 2001a), and that the book-to-market effect mean-reverts at about the same rate (Nagel, 2001). The benchmark correlation with the pricing kernel is $-\rho$ or -0.70. This value implies that 49% of the variation in each firm's revenues is systematic. The benchmark value of ρ seems high in absolute terms relative to studies such as Vuolteenaho (2002). We could achieve the same results by reducing ρ and increasing the maximal annual Sharpe Ratio proportionately. For example, our cross-sectional moments change little if we increase the maximum achievable Sharpe Ratio from 0.50 to 0.75 and reduce ρ from 0.70 to 0.467. The systematic variation in each firm's revenues is then 22%.

Enhanced momentum profits: Table 3, Panel C shows that, indeed, our numerical firms do produce enhanced momentum profits. For example, implementing a momentum strategy in $\sigma = 0.5$ firms produces quarterly profits of 1.92%, which is higher than the unconditional (numerical) momentum profits of 1.64% reported in Panel B, and $\sigma = 0.1$ firms produce quarterly profits of only 0.94%, which is 0.98% less than the profits from the high-σ firms. We see similar patterns when forming portfolios with low cost and high market-to-book firms.[15] We calculate enhanced momentum profits in numerical firms in the same manner as we do with CRSP/Compustat firms. A full description of the empirical methodology is given in Section 4.1.

Our numerical analysis also produces higher momentum profits in up markets (3.09%) than down markets (0.20%). This result makes intuitive sense. During an up market all firms move closer to regions of relatively higher log convexity. During a down market, all firms move closer to their endogenous shutdown points and regions of lower log convexity. Our results are close to the values reported in Cooper, Gutierrez, and Hameed (2004), who report quarterly momentum returns in up markets of 2.82% and quarterly returns in down markets of −1.11%, for a difference of 3.93%. The quarterly difference in our numerical results is 2.89%. These results are also consistent with the findings of Chordia and Shivakumar (2002).

4. Empirical tests and enhanced momentum strategies

Direct tests of our model require direct measures of the functional relation between log firm value and factor price—as shown in Figs. 1 and 2. But this functional relationship is not an observable firm attribute. Some indirect measures, such as CAPM betas, hinge on a specific asset pricing theory. Measuring CAPM betas of a firm immediately before and after a price move is very difficult and beta tends to perform poorly as a cross-sectional determinant of expected returns. To overcome these obstacles, we separate firms into those that our model predicts will exhibit positive return autocorrelation and those that it predicts will not. This separation produces enhanced momentum strategies by selecting only winners and losers that are expected to exhibit positive return autocorrelation.

4.1. Data

We obtain quarterly revenues, cost of goods sold, and book value of equity from the CRSP/Compustat merged data set for all available companies over the time period 1963:Q1 to 2004:Q3. We would prefer monthly firm data in order to match existing studies of momentum, but are limited by the quarterly reporting requirements in the United States. From the CRSP monthly data set, we extract monthly returns, stock prices, shares outstanding, and industry classification for all available companies from 1963:Jan to 2004:Dec. We use the monthly returns to calculate quarterly returns for each company.

[15] The high (low) market-to-book firms correspond to firms above (below) the 30th percentile.

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The market value of equity is calculated by multiplying stock price by the number of shares outstanding at the end of each quarter.

4.2. Conditional double-sort methodology

Each quarter, firms are ranked according to a firm-specific attribute called the "first criterion." We consider three different firm-specific attributes in this paper: revenue growth volatility, costs, and market-to-book. The ranking assigns firms to one of four quartiles. We consider the first criterion to be "high" if a firm is in the top "P" quartiles (either top quartile or top two quartiles). We consider the first criterion to be "low" if a firm is in the bottom "P" quartiles (either bottom quartile or bottom two quartiles).

Within each quartile, firms are ranked from highest to lowest according to their last quarter's returns. Firms are divided into twenty bins based on this second criterion ranking (within each quartile). The holding period (future) return for each of the eighty bins is calculated as the value-weighted return of the firms in the respective bin. Our choice of twenty bins within each quartile is similar to Moskowitz and Grinblatt (1999). We require eighty stocks per quarter. By value-weighting within a given bin, we ensure our results are not overly influenced by small-firms. Readers who are familiar with the momentum literature can consider our portfolios to be based on one-quarter formation periods with one-quarter holding periods.

Fig. 4 depicts the conditional double sort methodology. The figure shows a $P = 1, N = 3$ sorting when the market-to-book ratio is used as the first criterion. Consider high

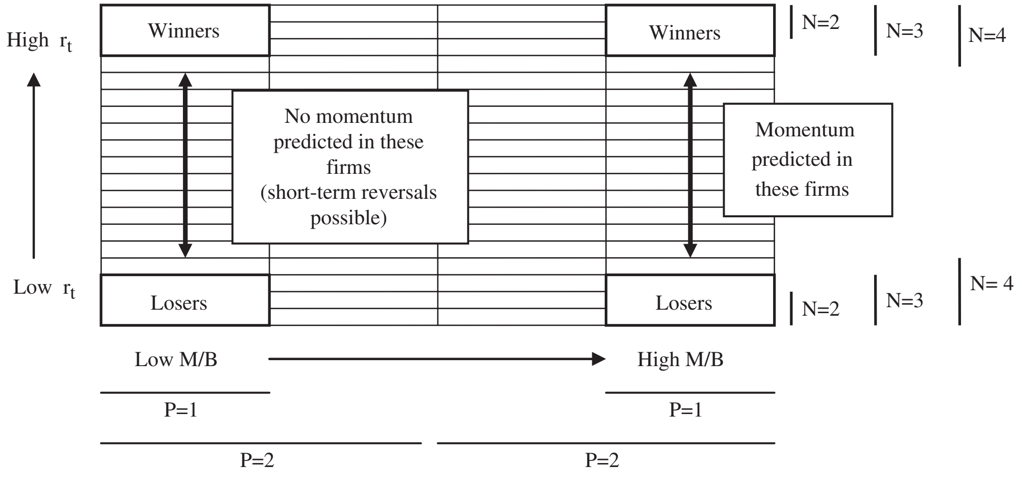

Fig. 4. Conditional double-sort procedure. The figure graphically depicts the conditional double-sort procedure used in this paper. We first sort firms into quartiles by an observable firm-specific attribute (such as the firm's market-to-book ratio). Within each quartile, we then sort firms into 20 bins based on current returns. We then compare the returns to a momentum strategy implemented in one set of firms (e.g., high M/B) to a momentum strategy implemented in the complementary set of firms (e.g., low M/B). When $P = 1$ we compare the strategies in the top quartile with strategies in the bottom quartile. When $P = 2$, we compare strategies in the top two quartiles with strategies in the bottom two quartiles. When $N = 2$, we use firms in the top two and bottom two returns bins. When $N = 3$, we use firms in the top three and bottom three return bins. When $N = 4$, we use firms in the top four and bottom four returns bins.

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market-to-book firms only (those in the top quartile). Momentum profits equal the return of the top three bins (winners) minus the return of the bottom three bins (losers.) One of our goals is to measure the difference between momentum profits from the high market-to-book firms and the momentum profits from the low market-to-book firms.

Existing studies of momentum typically rely on a single-sort methodology and use returns as the sole sorting criterion. However, three well-known studies employ a double-sort methodology: Lakonishok, Shleifer, and Vishny (1994), Chan, Jegadeesh, and Lakonishok (1996), and Lee and Swaminathan (2000). A key difference between our study and existing work is that the three aforementioned studies employ independent double sorts whereas we use conditional double sorts. The conditional double-sort methodology allows us to compare similar momentum strategies (and profits) after conditioning on other variables. Conditional double sorts also ensure an equal number of firms in each bin. Independent double sorts, on the other hand, can end up with many firms in some bins and few firms in other bins if the two sorting variables are correlated.

4.3. Return autocorrelation

We begin by confirming that, as in the studies of Jegadeesh and Titman (1993), Rouwenhorst (1998, 1999), and Jegadeesh and Titman (2001b), momentum is present in our sample of quarterly data. Table 4 shows the results of a single-sort procedure. In Panel A, firms are ranked by current returns (quarter t) and sorted into bins. The future, one-quarter return (over quarter $t+1$) of a portfolio long the top N bins and short the bottom N bins is shown as the holding period return. We annualize this number for convenience. Panel A shows that winners in the top three bins ($N=3$ or top 15%) outperform losers by 1.94% per quarter or 7.98% per annum. This result is statistically significant ($t=2.05$). Momentum profits from a portfolio long the top two bins and short the bottom two bins ($N=2$) correspond to returns of 2.66% per quarter or 11.08% per annum ($t=2.56$).

Table 4, Panel B reports returns to momentum portfolios restricted to firms that possess the accounting data required in our double sorts. We use the results in Panel B to test for enhanced momentum strategies and we discuss them more thoroughly below.

4.4. Revenue growth volatility

We construct our measure of revenue growth volatility by first calculating date t revenue growth:

$$g_t^{\text{rev}} = \frac{revenues_t - revenues_{t-4}}{revenues_{t-4}}. \tag{24}$$

Note that Eq. (24) accounts for potential seasonality in revenues because our data are quarterly. We calculate revenue growth volatility for quarter t as the standard deviation of the most recent ten observations $g_{t-9}^{\text{rev}}, \ldots, g_t^{\text{rev}}$. We require at least five observations between $t-9$ and t. Table 5 presents the results of the double sort (as described earlier) with a firm's revenue volatility as the first sorting criterion. The results are broadly consistent with the simulation results. Focusing on the $P=1, N=4$ strategy (bottom and top revenue volatility quartiles, top/bottom four bins), the only discernible momentum profits correspond to the high revenue volatility quartile (2.39% versus −0.77%). The

Table 4
Momentum strategy returns

The momentum portfolios are formed by employing a single-sort methodology of returns at the firm level. Each quarter, from 1963:Q1 to 2004:Q3, firms are ranked by past returns in descending order. Firms are then divided into twenty bins based on this ranking (each bin contains 5% of the stocks). The holding period return for each bin is calculated as the value-weighted return of firms in the bin. The average return of the top N bins (“winners”) minus the average return of the lowest N bins (“losers”) is the called the return to the momentum portfolio (“W – L”). Panel A shows momentum strategy returns using all available firms. Panel B shows momentum strategies using firms that also have accounting data available (revenues, costs, and market to book ratios).

Panel A: Momentum strategies

P	N	Sort #1: Recent stock returns	Sort #2: n.a	Holding period return	Annual return	t-start	# of qtrs
n.a	2	Top & bottom 2 bins	n.a	0.0266	0.1108	2.56	166
n.a	3	Top & bottom 3 bins	n.a	0.0194	0.0798	2.05	166
n.a	4	Top & bottom 4 bins	n.a	0.0155	0.0633	1.82	166

Panel B: Momentum strategies after screening firms

Revenue volatility screen			Cost screen			M/B screen		
N	Holding Period Return	t-stat	N	Holding Period return	t-stat	N	Holding Period return	t-stat
2	0.0186	1.74	2	0.0304	2.43	2	0.0269	2.32
3	0.0141	1.46	3	0.0235	2.14	3	0.0184	1.74
4	0.0106	1.23	4	0.0192	1.97	4	0.0153	1.61

difference between the profitability of the two strategies is highly significant ($t = 3.79$) and economically large (13.26% in annual terms).

In Table 5, Panel B, we account for the fact that quarter $t-1$ accounting information is typically announced during quarter t. We calculate lagged revenue growth volatility for quarter t as the standard deviation of the $g^{\text{rev}}_{t-10}, \ldots, g^{\text{rev}}_{t-1}$. Again, we require at least five observations between $t-10$ and $t-1$. Focusing again on the $P = 1, N = 4$ strategy (bottom and top revenue volatility quartiles, top/bottom four return bins), the only discernible momentum profits correspond to the high revenue volatility quartile (1.91% versus −0.77%). The difference between the profitability of the two strategies is highly significant ($t = 3.38$) and remains economically large (11.14% in annual terms).

We see that a momentum strategy implemented in the high volatility firms outperforms a similar strategy implemented in all firms. The comparison is made by considering Table 4, Panel B returns under “Revenue Volatility Screen.” These returns are generated after requiring five of the past ten quarters to have revenue volatility data (as in Table 5). When comparing Table 5, Panel A and Table 4, Panel B, we see the $P = 1, N = 4$ enhanced momentum strategy implemented in high volatility firms greatly outperforms a traditional strategy by 1.33% = 0.0239 – 0.0106 per quarter. This outperformance in returns is equal to 5.43% on an annual basis. Similar outperformance obtains for other bins as well. The

Table 5
Revenue growth volatility

The momentum portfolios are formed by employing a double-sort methodology at the firm level. Theory predicts that momentum portfolios (long past winners and short past losers) formed from high volatility firms will outperform momentum portfolios formed from low volatility firms. We first sort firms into revenue volatility quartiles each quarter from 1963:Q1 to 2004:Q3. Then, within each quartile group, we implement a momentum strategy by sorting stocks into twenty bins based on past returns and forming a portfolio that is long the winners and short the losers. Past winners are defined as stocks in the top N bins. Past losers are defined as stocks in the lowest N bins. "High volatility" refers to the top P quartiles, while "low volatility" refers to the bottom P quartiles. Panel B takes into account that quarter t sales are not officially announced until some point in quarter $t+1$. Therefore, sales are lagged by an additional quarter. The holding period is three months–over quarter $t+1$.

Panel A: Revenue growth volatility

P	N	Sort #1: Lagged revenue growth volatility	Sort #2: Recent returns	High volatility W − L	Low volatility W − L	Diff. in returns	Annual diff. in returns	t-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0290	−0.0053	0.0343	0.1444	3.20	158
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0269	−0.0056	0.0325	0.1363	3.48	158
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0239	−0.0077	0.0316	0.1326	3.79	158
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0270	0.0019	0.0251	0.1042	3.46	158
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0240	0.0005	0.0235	0.0972	3.76	158
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0203	−0.0012	0.0215	0.0887	3.83	158

Panel B: Lagged revenue growth volatility

P	N	Sort #1: Lagged revenue growth volatility	Sort #2: Recent returns	High volatility W − L	Low volatility W − L	Diff. in returns	Annual diff. in returns	t-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0238	−0.0043	0.0281	0.1170	2.86	157
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0188	−0.0044	0.0232	0.0960	2.59	157
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0191	−0.0077	0.0268	0.1114	3.38	157
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0253	0.0028	0.0225	0.0929	3.18	157
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0185	0.0029	0.0156	0.0638	2.43	157
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0189	0.0007	0.0182	0.0747	3.28	157

higher momentum returns are the result of an enhanced momentum strategy that yields an additional 0.44% to 1.33% per quarter or 1.77% to 5.43% per year.

4.5. Costs

Our model and numerical analysis both indicate that costs decrease return autocorrelation. We focus on costs borne by equity holders. The test employs the double-sort methodology using costs of goods sold divided by total assets as the first criterion. Table 6 presents our results, with Panel A sorting by the most recent quarter's costs and Panel B sorting by lagged costs to account for reporting delays. Consider Panel B: $P=1, N=4$. A momentum strategy implemented in only low cost firms realizes an average return of 2.79% per quarter. A momentum strategy implemented in only high cost firms realizes an

Table 6
Costs

The momentum portfolios are formed by employing a double-sort methodology at the firm level. Theory predicts that momentum portfolios (long past winners and short past losers) formed from low cost firms will outperform momentum portfolios formed from high costs firms. We first sort firm into cost quartiles each quarter from 1963:Q1 to 2004:Q3. We use cost of goods sold divided by total assets. Then, within each quartile group, we implement a momentum strategy by sorting stocks into twenty bins based on past returns and forming a portfolio that is long the winners and short the losers. Past winners are defined as stocks in the top N bins. Past losers are defined as stocks in the lowest N bins. "High costs" refers to the top P quartiles, while "low costs" refers to the bottom P quartiles. Panel B takes into account that quarter t costs are not officially announced until some point in quarter $t+1$. Therefore, costs are lagged by an additional quarter. The holding period is three months—over quarter $t+1$.

Panel A: Costs

P	*N*	Sort #1: Costs	Sort #2: Recent returns	Low cost W – L	High cost W – L	Diff. in returns	Annual diff. in returns	*t*-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0296	0.0303	−0.0007	−0.0029	−0.07	128
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0253	0.0201	0.0051	0.0208	0.51	128
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0202	0.0119	0.0083	0.0336	0.98	128
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0327	0.0182	0.0145	0.0592	2.04	128
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0282	0.0097	0.0185	0.0761	3.07	128
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0230	0.0044	0.0186	0.0763	3.64	128

Panel B: Lagged costs

P	*N*	Sort #1: Costs	Sort #2: Recent returns	Low cost W – L	High cost W – L	Diff. in returns	Annual diff. in returns	*t*-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0419	0.0207	0.0213	0.0878	1.73	127
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0338	0.0122	0.0216	0.0894	2.11	127
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0279	0.0082	0.0198	0.0814	2.29	127
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0309	0.0152	0.0157	0.0642	1.88	127
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0268	0.0068	0.0201	0.0827	3.03	127
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0227	0.0022	0.0205	0.0846	3.57	127

average return of 0.82% per quarter. The difference between these two momentum strategies is 1.98% per quarter or 8.14% per annum and is statistically significant at conventional levels. Strategies implemented in other quartiles yield profits of similar magnitude. Statistical significance becomes marginal when we use only the bottom quartile and top quartile ($P=1$) in Panel A. This could be due to the fact that cost data are missing for a number of firms in our data. Reducing the population in each of the bins inevitably leads to an increase in measurement error, and this effect is most keenly felt in the strategies that use relatively few bins.

Of our three empirical double sorts, the enhanced momentum effect is least strong when considering costs. We compare results in Table 4, Panel B with results in Table 4, Panel B. For $P=1, N=2$ the enhanced strategy yields $1.15\% = 4.19\% - 3.04\%$ per quarter more than a traditional sort from Table 4. For $P=1, N=3$ the enhanced strategy yields $1.03\% = 3.38\% - 2.35\%$ per quarter more than a traditional sort.

Does our double-sort methodology with costs effectively sort on cross-industry cost structure as in Moskowitz and Grinblatt (1999)? The answer is no. Consider an industry with low costs. Firms from this industry are more likely to end up in the lowest cost quartile than in the highest quartile. We show that momentum is strongest in this lowest quartile. Thus, in the second sort, we are sorting firms on an *intra-industry* basis, whereas Moskowitz and Grinblatt (1999) sort firms *by industry*. The stories in our paper and the previous work are therefore different from each other.

Are the results in Table 6 obviously due to the "leverage effect?" Our paper suggests that a decrease in the output price of a firm's good might lead to an increase in expected returns in post-exercise firms. However, we also show that the same situation can lead to decreased expected returns (momentum) in pre-exercise firms. The sign of the change in expected returns is by no means obvious and depends on the level of costs and the magnitude of growth options.

4.6. Market-to-book

Table 7 confirms that momentum portfolios formed from high market-to-book firms significantly outperform momentum portfolios formed from low market-to-book firms. For example, if we look at the results from Panel A's $P=1, N=4$ strategy, we see that high market-to-book firms have an average return to momentum investing of 3.44% per quarter, whereas low market-to-book firms have an average return of 1.01% per quarter. The difference of 2.43% per quarter or 10.08% per annum is both statistically and economically significant ($t=2.18$). The results from other combinations of quartiles and bins show a similar pattern. As with the revenue volatility double sort, the majority of momentum profits come from firms we expect have high return autocorrelation. Low market-to-book firms exhibit much lower momentum profitability.

At first glance, some readers may confuse our use of a firm's market-to-book ratio with the Fama and French (1992) book-to-market factor. There is a difference. Fama and French (1992) show that high book-to-market firms outperform low book-to-market firms. Their results pertain to unconditional returns. While these results also hold in our numerical analysis (and in our data), the results in Table 7 pertain to expected returns after conditioning on past returns. Our results show that high market-to-book firms have higher expected returns after a positive return. The same firms have lower expected returns after a negative return. In most respects, our results are orthogonal to those of Fama and French (1992).

Finally, Table 7 and Table 4, Panel B show that enhanced momentum strategies using high market-to-book firms outperform traditional strategies by up to 2.19% per quarter or 9.05% per year.

4.7. Other tests

We test whether the double-sort methodology employed in Tables 5–7 inadvertently sorts by firm size. We repeat the double-sort methodology by first sorting firms into quartiles based on total book value of assets. Within each quartile we form momentum

Table 7
Growth options

The momentum portfolios are formed by employing a double-sort methodology at the firm level. Theory predicts that momentum portfolios (long past winners and short past losers) formed from high-market-to-book firms will outperform momentum portfolios formed from low market-to-book firms. We first sort firm into market-to-book quartiles each quarter from 1963:Q1 to 2004:Q3. Then, within each quartile group, we implement a momentum strategy by: (i) sorting stocks into twenty bins based on past returns; and (ii) forming a portfolio that is long the winners and short the losers. Past winners are defined as stocks in the top N bins. Past losers are defined as stocks in the lowest N bins. ''High market-to-book'' refers to the top P market-to-book quartiles, while ''low market-to-book'' refers to the bottom P quartiles. This table takes into account that quarter t market-to-book is not officially announced until some point in quarter $t+1$. Therefore, market-to-book is lagged by an additional quarter. The holding period is three months—over quarter $t+1$.

Panel A: Market-to-book

P	N	Sort #1: Market-to-book	Sort #2: Market-to-book	High M-to-B W – L	Low M-to-B W – L	Diff. in returns	Annual diff. in returns	t-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0422	0.0102	0.0321	0.1346	2.29	136
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0390	0.0093	0.0297	0.1240	2.35	136
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0344	0.0101	0.0243	0.1008	2.18	136
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0350	0.0080	0.0271	0.1127	2.94	136
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0311	0.0052	0.0259	0.1076	3.13	136
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0258	0.0047	0.0210	0.0868	2.86	136

Panel B: Lagged market-to-book

P	N	Sort #1: Market-to-book	Sort #2: Market-to-book	High M-to-B W – L	Low M-to-B W – L	Diff. in returns	Annual diff. in returns	t-stat	# of qtrs
1	2	Top & bottom quartile	Top & bottom 2 bins	0.0396	0.0167	0.0228	0.0945	1.70	135
1	3	Top & bottom quartile	Top & bottom 3 bins	0.0383	0.0124	0.0259	0.1078	2.25	135
1	4	Top & bottom quartile	Top & bottom 4 bins	0.0372	0.0098	0.0274	0.1141	2.62	135
2	2	Top & bottom 2 quartiles	Top & bottom 2 bins	0.0259	0.0118	0.0141	0.0577	1.55	135
2	3	Top & bottom 2 quartiles	Top & bottom 3 bins	0.0252	0.0079	0.0173	0.0711	2.26	135
2	4	Top & bottom 2 quartiles	Top & bottom 4 bins	0.0226	0.0055	0.0171	0.0701	2.44	135

portfolios. We find no significant difference between momentum strategy returns. When sorting on current book value, high book value firms produce momentum strategy profits of 0.45% per quarter using a $P=1, N=2$ strategy. Low book value firms produce profits of 0.68%. The difference is −0.23% per quarter with a t-statistic of −0.22. When sorting on *lagged* book value, high book value firms produce momentum strategy profits of 0.61% per quarter using a $P=1, N=2$ strategy. Low book value firms produce profits of 1.03%. The difference is −0.42% per quarter, with a t-statistic of −0.43.

We repeat the double-sort methodology by first sorting firms into quartiles based on the level of revenues (as opposed to revenue growth volatility as in Table 5). Within each

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quartile we form momentum portfolios. Again, we find no significant difference between momentum strategy returns. When sorting on current revenues, high revenue firms produce momentum strategy profits of 0.77% per quarter using a $P=1, N=2$ strategy. Low revenue firms produce profits of 0.69%. The difference is 0.08% per quarter, with a t-statistic of 0.10. When sorting on *lagged* revenues, high revenue firms produce momentum strategy profits of 0.56% per quarter using a $P=1, N=2$ strategy. Low revenue firms produce profits of 0.28%. The difference is 0.28% per quarter, with a t-statistic of 0.32.

5. Conclusion

This paper contributes to our understanding of stock price returns in a number of ways. We show that one can condition on past returns and observable firm-specific attributes to learn about future expected returns. We conduct our analysis in three integrated steps. First, we provide a model of firms with revenues, costs, growth options, and shutdown options. We solve for the conditions when one would expect a firm to exhibit positive return autocorrelation. We show that operating leverage reduces return autocorrelation and extreme operating leverage leads firms to have negative return autocorrelation. Growth and limited liability options that do not expire increase return autocorrelation.

Second, we run a numerical analysis of our model firm. We are able to calculate the sensitivity of return autocorrelation to each of the model parameters. When considering observable firm attributes, we show that return autocorrelation is increasing in return volatility, decreasing in costs, and increasing in the market-to-book ratio. We are also able to construct a population of model firms. These firms allow us to study momentum strategy profits. We parameterize our firms so as to roughly target the historic market risk premium, size premium, value premium, and momentum premium. Our model firms produce profits from enhanced momentum strategies. That is, momentum strategies carried out in high revenue volatility firms, low cost firms, and high market-to-book firms all produce greater profits than a traditional Jegadeesh and Titman (1993) strategy. Finally, our model firms exhibit higher momentum profits in up markets than they do in down markets.

The third and final step in our analysis is to test the implications of our model with data. Using a conditional double-sort methodology and CRSP/Compustat firms, we construct enhanced momentum strategies by first sorting on high revenue volatility firms, low cost firms, and high market-to-book firms. Our strategies produce momentum profits that outperform traditional strategies by approximately 5%/year. The momentum strategies implemented in CRSP/Compustat firms produce profits of the same sign and magnitude as momentum strategies implemented in our model firms.

Recent literature shows that the functional relation between firm value and cash flow variables is an important determinant of conditional expected returns. We build on this finding and identify proxies that are empirically relevant when determining which firms might exhibit positive return autocorrelation and which firms might not. Our predictions (and their empirical confirmation) concerning momentum profits conditioned on costs and revenue volatility are, to our knowledge, new. The predictions of our model and our results shed light on the anatomy of momentum strategies and, more generally, the cross-section of returns. This paper offers a single framework for understanding existing momentum profits as well as profits from new, enhanced momentum strategies.

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Appendix A. The real options model

A.1. Return autocorrelation in a continuous time diffusion economy

Assume the underlying uncertainty in the economy corresponds to a multivariate Itô diffusion and an idiosyncratic Poisson death process with constant arrival rate λ. The ex-dividend market value of a firm's equity (V_t) is equal to the maximum present value attainable from the management of cash flows and satisfies a set of differential equations:

$$\frac{1}{2}\mathrm{Tr}\left[\boldsymbol{\sigma}_t'\frac{\partial^2 V_t}{\partial \boldsymbol{X}_t \partial \boldsymbol{X}_t'}\boldsymbol{\sigma}_t\right] + \boldsymbol{\mu}_t^{*\prime}\cdot\frac{\partial V_t}{\partial \boldsymbol{X}_t} - (r_t+\lambda)V_t + c_t^* = 0, \tag{25}$$

$$\mathrm{d}\boldsymbol{X}_t = \boldsymbol{\mu}_t^*\,\mathrm{d}t + \boldsymbol{\sigma}_t'\,\mathrm{d}\boldsymbol{W}_t. \tag{26}$$

Here, $\boldsymbol{X}_t$ is a vector of $f = 1, \ldots, F$ "factors," $\boldsymbol{\mu}_t^*$ is a vector of risk-adjusted growth rates, c_t^* is an optimal choice of cash flows given the operating constraints of the firm,[16] r_t is the prevailing risk-free rate, and $\boldsymbol{W}_t$ is a vector of Brownian variates. The date t expected total instantaneous rate of return of the firm is

$$\text{Total Rate of Return}_t \cdot \mathrm{d}t = \frac{E_t[\mathrm{d}V_t] + c_t^*\,\mathrm{d}t}{V_t}.$$

From Itô's Lemma, this becomes

$$\text{Total Return}_t = \left\{\frac{1}{2}\mathrm{Tr}\left[\boldsymbol{\sigma}_t'\frac{\partial^2 V_t}{\partial \boldsymbol{X}_t\,\partial \boldsymbol{X}_t'}\boldsymbol{\sigma}_t\right] + \boldsymbol{\mu}_t'\cdot\frac{\partial V_t}{\partial \boldsymbol{X}_t} - \lambda V_t + c_t^*\right\}\frac{1}{V_t}, \tag{27}$$

where $\boldsymbol{\mu}_t$ is the unadjusted drift of $\boldsymbol{X}_t$. Using (25), we can simplify Eq. (27) to

$$\text{Total Return}_t = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)'\cdot\frac{\partial v_t}{\partial \boldsymbol{X}_t} + r_t, \tag{28}$$

where $v_t \equiv \ln V_t$. Eq. (28) can also be derived more intuitively by considering a firm that is a portfolio of factor-mimicking assets that correspond to the risk-free asset and $n>0$ assets that have log price X_t^n.

A.2. Proofs

Proof of Proposition 1. Let $\boldsymbol{D}v_t$ be the instantaneous change in $v_t \equiv \ln V_t$. The corresponding change in expected *excess* returns is $\boldsymbol{D}((\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)'\cdot\partial v_t/\partial \boldsymbol{X}_t)$. By Itô's Lemma, the covariance between the latter expression and changes to v_t is

$$\boldsymbol{D}\left((\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)'\cdot\frac{\partial v_t}{\partial \boldsymbol{X}_t}\right)\cdot\boldsymbol{D}v_t = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_t^*)'\cdot\frac{\partial^2 v_t}{\partial \boldsymbol{X}_t\,\partial \boldsymbol{X}_t'}\boldsymbol{\sigma}_t\boldsymbol{\sigma}_t'\cdot\frac{\partial v_t}{\partial \boldsymbol{X}_t}.$$

The sensitivity of returns to changes in v_t is defined to be the instantaneous covariance between v_t and excess returns (the last expression) divided by the variance of changes in v_t (i.e., $\partial v_t/\partial \boldsymbol{X}_t'\cdot\boldsymbol{\sigma}_t\boldsymbol{\sigma}_t'\cdot\partial v_t/\partial \boldsymbol{X}_t$). □

[16] Embedded in the cash flows are various endogenous decisions (e.g., to expand or to shut down production) contingent on realizations of the macroeconomic variables, $\boldsymbol{X}_t$. Consequently, V_t must also satisfy a set of Bellman equations and smooth pasting conditions. See, for instance, Brennan and Schwartz (1985) and Brekke and Øksendal (1991).

Proof of Proposition 2. Letting $v_t^B \equiv \ln V_t^B$, the factor loading or firm beta is, according to the equations in the text,

$$\beta(p_t) \equiv \frac{\partial v_t^B}{\partial p_t} = \int_0^1 \eta(p_t, s) s \, \mathrm{d}s, \tag{29}$$

where

$$\eta(p_t, s) = \frac{1}{\theta} \frac{s^{(r+\lambda)/\theta - 1} e^{p_t s + (1-s)\mu^*/\theta + \sigma^2(1-s^2)/4\theta}}{V_t}.$$

Note that $\eta(p_t, s)$ is a probability measure (it is everywhere positive and its integral over $s \in [0, 1]$ is one) so $\partial v_t^B / \partial p_t$ is always positive. This immediately implies that the beta of the cash flows present value (with respect to the priced factor, p_t) is between zero and one. Therefore,

$$\frac{\partial^2 v_t^B}{\partial^2 p_t} = \int_0^1 \eta(p_t, s) s^2 \, \mathrm{d}s - \left(\int_0^1 \eta(p_t, s) s \, \mathrm{d}s \right)^2,$$

which is the variance of the distribution defined by $\eta(p_t, s)$, and therefore positive. □

Proof of Lemma 2.1. Assume that $\ln V_1$ and $\ln V_2$ are convex in p. Then

$$\begin{aligned}
\frac{\partial^2}{\partial p^2} \ln(V_1 + V_2) &= \frac{\partial}{\partial p} \frac{V_1' + V_2'}{V_1 + V_2} \\
&= \frac{\partial}{\partial p} \left(\frac{V_1}{V_1 + V_2} \frac{V_1'}{V_1} + \frac{V_2}{V_1 + V_2} \frac{V_2'}{V_2} \right) \\
&= \frac{V_1}{V_1 + V_2} \frac{\partial^2}{\partial p^2} \ln(V_1) + \frac{V_2}{V_1 + V_2} \frac{\partial^2}{\partial p^2} \ln(V_2) \\
&\quad + \left(\frac{V_1'}{V_1} - \frac{V_2'}{V_2} \right) \frac{\partial}{\partial p} \frac{V_1}{V_1 + V_2}.
\end{aligned}$$

The last term can be written as

$$\frac{\partial}{\partial p} \ln\left(\frac{V_1}{V_2} \right) \frac{\partial}{\partial p} \frac{V_1/V_2}{1 + V_1/V_2} = \frac{V_2/V_1}{(1 + V_1/V_2)^2} \left(\frac{\partial}{\partial p} \left(\frac{V_1}{V_2} \right) \right)^2 > 0.$$

The result of the lemma follows. □

Proof of Proposition 3. The generalized Hermite function can be written in the limit form,

$$H(-a, z) = \lim_{v \to \infty} 2^{-a/2} \Gamma(1-a) v^{a/2} L_{-a}^{v}(v - \sqrt{2v}z),$$

where $\Gamma(\cdot)$ is the Gamma function and $L_{-a}^{v}(\cdot)$ is the generalized Laguerre function,

$$L_{-a}^{v}(x) = \frac{\sin(a\pi)}{\pi} \int_0^1 e^{xt} t^{a-1} (1-t)^{v-1} \, \mathrm{d}t, \quad a > 0. \tag{30}$$

For each v, one can use a similar argument as used in establishing the convexity of v_t^B to show that the log of $2^{-a/2}\Gamma(1-a)v^{a/2}L_{-a}^{v}(v - \sqrt{2v}z)$ is convex in z. The log of the limit expression, $\ln H(-a,z)$, must therefore also be convex. The above is true for any finite and positive a, thus $\ln H(r/\theta, \pm(\mu/\theta - p)\sqrt{\theta}/\sigma)$ is strictly convex in p if and only if $\theta > 0$. Because $\alpha_\pm \geqslant 0$, an immediate implication of Lemma 1 is that $\ln(\alpha_+(U_+(p_t))/(U_+(\bar{p})) + \alpha_-(U_-(p_t))/(U_-(\underline{p})))$ is also convex for every $\theta \geqslant 0$.

According to Proposition 1 and Eqs. (4a), the instantaneous return autocorrelation from holding an asset with value V is proportional to $\frac{C_V}{\beta_V}$, where for any $x > 0$ we define $\partial^2 \ln x/\partial p^2 \equiv C_x$ and $\frac{\partial \ln x}{\partial p} \equiv \beta_x$. Suppose $x, y > 0$ and $\beta_x, \beta_{x+y} > 0$. Then Lemma 1 can be used to show that

$$\frac{C_{x+y}}{\beta_{x+y}} > \frac{C_x}{\beta_x} \Leftrightarrow \beta_x C_y - \beta_y C_x + \beta_x \frac{x}{x+y}(\beta_y - \beta_x)^2 > 0. \tag{31}$$

Finally, define $V_1 \equiv V^B(p_t)$, $V_2 \equiv K/(r+\lambda)$, and $V_3 \equiv \alpha_+ U_+(p_t)/U_+(\bar{p}) + \alpha_- U_-(p_t)/U_-(\underline{p})$. It follows from the previous paragraph, the hypothesis, and Proposition 2 that $C_{V_3} \geqslant 0$, $C_{V_1 - V_2} < 0$, and $C_{V_1} \geqslant 0$. It is also the case that $\beta_{V_1 - V_2} > 0$.

Suppose, that $\beta_{V_3} \geqslant 0$. To prove the proposition, it is sufficient to establish that $C_{V_1 - V_2 + V_3}/\beta_{V_1 - V_2 + V_3} \geqslant C_{V_1 - V_2}/\beta_{V_1 - V_2}$. This follows immediately from Eq. (31) with the identification: $x = V_1 - V_2$ and $y = V_3$.

Next, suppose that $\beta_{V_3} < 0$. Then $C_{V_1 - V_2 + V_3}/\beta_{V_1 - V_2 + V_3} = C_{V_1 + V_3}/\beta_{V_1 + V_3} - (V_2/V_1 - V_2 + V_3)\beta_{V_1 + V_3}$. To prove the result, it is sufficient to show that $C_{V_1 + V_3}/\beta_{V_1 + V_3} - V_2/V_1 - V_2 + V_3\beta_{V_1 + V_3} > C_{V_1}/\beta_{V_1} - V_2/V_1 - V_2\beta_{V_1}$. Since $\beta_{V_1 + V_3} < \beta_{V_1}$ and $V_3 > 0$, it is sufficient to demonstrate that $C_{V_1 + V_3} \geqslant C_{V_1}$. This too follows from Eq. (31), by setting $y = V_3$ and $x = V_1$, and using Proposition 2. □

Appendix B. Limitations of the single-factor approach and the effect of idiosyncratic risk

When a firm's returns depend on only a single stochastic variable (p_t), the effect of idiosyncratic risk is buried in the volatility parameter of p_t and in the risk premium, $\mu - \mu^*$.[17] Consequently, in a one-factor setting idiosyncratic risk does not enter directly into the determination of return autocorrelation—see Eq. (4a). Consider applying Proposition 1 to a portfolio whose value is $V_t = ae^{X_t^{n1}} + be^{X_t^{n2}}$. Here, X_t^{n1} and X_t^{n2} are distinct sources of risk and $\{a, b\}$ are constants. Assume that $r_{n_1} = \mu_{n_1} - \mu^*_{n_1}$ and $r_{n_2} = \mu_{n_2} - \mu^*_{n2}$ are constant factor risk premia. Application of Proposition 1 shows that the sensitivity of expected returns to an unexpected change in log firm value is

$$(r_{n_1} - r_{n_2})\omega(1-\omega)\frac{\omega\sigma_{n_1}^2 - (1-\omega)\sigma_{n_2}^2 + (1-2\omega)\rho\sigma_{n_1}\sigma_{n_2}}{\omega^2\sigma_{n_1}^2 - (1-\omega)^2\sigma_{n_2}^2 + \omega(1-\omega)\rho\sigma_{n_1}\sigma_{n_2}}, \tag{32}$$

where $\omega \equiv ae^{X_t^{n1}}/V_t$, $(\sigma_{n_1}, \sigma_{n_2})$ are the factor diffusion coefficients, and ρ is the instantaneous correlation between the two processes. Since the log convexity of X_t^{n1} or X_t^{n2} individually is zero, if either one is risk-free then the result reduces to Lemma 1. Suppose, on the other hand, that both assets are risky yet uncorrelated ($\rho = 0$), and that X_t^{n2} has zero risk premium ($r_{n_2} = 0$). If $\sigma_{n_2}/\sigma_{n_1}$ is smaller than one, then depending on the relative weights the qualitative message from Lemma 1 is still valid. However, if $\sigma_{n_2}/\sigma_{n_1}$ is

[17] The Sharpe Ratio associated with the factor p_t is $(\mu - \mu^*)/\sigma$; the ratio of this Sharpe Ratio to the maximal Sharpe Ratio is the negative correlation between the factor p_t and the state-price deflator. Thus $(\mu - \mu^*)/\sigma$ is a measure of the systematic risk inherent in p_t.

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sufficiently large, the sensitivity can be negative even when the portfolio is long both assets. Thus, a firm whose expected returns are largely driven by a relatively less volatile part of its portfolio of assets should not exhibit positive return autocorrelation. For instance, if the idiosyncratic volatility σ_{n_2} is twice as high as σ_{n_1}, then the firm will exhibit positive return autocorrelation only if the weight of the "priced" assets in the portfolio is greater than 80%. It is important to emphasize that the separability of the assets is crucial in the previous conclusions. It is not the case that firms exhibit *decreasing* returns to value if the variation in firm value is largely explained by unpriced or idiosyncratic shocks. In a single-factor model idiosyncratic risk is a component of σ_{n_1} whenever $X_t^{n_1}$ is not perfectly correlated with the pricing kernel in the economy. Moreover, idiosyncratic risk can comprise almost all of the volatility and, holding the risk premium constant, this does not affect the return autocorrelation. Our analysis is not a complete examination of all the various possibilities in which idiosyncratic shocks can affect return autocorrelation. However, the analysis provides a caveat to the results in Section 2: the results are generally valid only to the extent that, in most firms, expected returns are largely driven by those assets with higher volatility.

Appendix C. Parameters matched to market data

We estimate parameter values for ξ, $Pr^{\text{pre}}(exercise)$, $Pr^{\text{pre}}(exit)$, $Pr^{\text{post}}(exit)$, and the ratio of $K^{i,pre}$ to $K^{i,post}$ from market data. In order to do this we need to classify all firms as either pre-exercise or post-exercise.

Our classification system uses physical size and the following assumptions as criteria: (i) the average post-exercise firm has an asset base that is ξ times larger than the average pre-exercise firm; (ii) the population of firms is in a steady state so that the proportion, n, of pre-exercise firms is constant; (iii) in the steady state, the post-exercise firms hold a constant share, x, of all assets; and (iv) the growth rate of total assets, g, held by all firms is also constant.

Our assumptions imply that $x = (1-n)\xi/[n + (1-n)\xi]$. Solving for ξ gives $\xi = \left(\frac{n}{1-n}\right)\left(\frac{x}{1-x}\right)$. We sort firms in our sample of Compustat firms in ascending order based on the book value of assets. Let $\hat{x}(\hat{n})$ be the total proportion of assets held by all firms larger than the $\hat{n}^{th}$ firm ($\hat{n} \in [0,1]$). The function $f(\hat{n}) \equiv (\frac{\hat{n}}{1-\hat{n}})(\frac{\hat{x}(\hat{n})}{1-\hat{x}(\hat{n})})$ can be viewed as an empirical proxy for ξ, given $\hat{n}$. Using data from 1980 to 2004, we produce the following:

$\hat{n}$ (%)	$1-\hat{x}(\hat{n})$(%)	$f(\hat{n})\sim\xi$	$\ln(\mathrm{d}\{1-\hat{x}(\hat{n})\}/\mathrm{d}\hat{n})$
65	4.6	38.2	−1.09
70	6.3	34.6	−0.67
75	8.9	30.8	−0.50
80	11.9	29.6	−0.06
85	16.6	28.4	0.38
90	24.0	28.6	1.01
95	37.7	31.4	2.52
100	100.0	NA	NA

The estimate for ξ in Column 3 does not vary much and all estimates are close to 30. One can view the second column as a cumulative distribution function for total assets. Column 4 gives the log of the implied density, which can be calculated by taking central differences.

The log of the implied density is roughly linear in $\hat{n}$ below the value of $\hat{n}=0.90$. The value of $\ln(\mathrm{d}\{1-\hat{x}(\hat{n})\}/\mathrm{d}\hat{n})$ at $\hat{n}=0.95$ is anomalously large. A linear regression of the first six points of $\ln(\mathrm{d}\{1-\hat{x}(\hat{n})\}/\mathrm{d}\hat{n})$ on $\hat{n}$ produces an adjusted-R^2 of 0.97 and the subsequent forecast of the log density at $\hat{n}=0.95$ is significantly below the 2.52 shown in the table. Including the last point ($\hat{n}=0.95$) in the linear regression reduces the R^2 to 0.88 and more than triples the standard error.

These regression results suggest that firms above $\hat{n}=0.90$ are significantly larger than those below $\hat{n}=0.90$. We therefore classify as pre-exercise firms all those that have asset values below the 90^{th} percentile. These conclusions are robust to the use of total sales as a measure of size instead of book assets.

Once pre-exercise and post-exercise firms are classified, we calculate various parameters. For instance, we find that $K^{\text{pre}} \approx K^{\text{post}}$ based on costs of goods sold (CGS). We therefore set these parameters equal. We do not fix K^{post} using data because there is no reliable accounting measure of cost that we can directly relate to K in our model. The average of costs of goods sold (CGS) over sales is 72% in market data and the average of CGS over total assets is 75%.

To calculate average exit rates, a firm is said to exit due to distress if its market value of equity falls 90% or more over a three-year period *and* the firm ceases to exist in the CRSP database over the following year. Our definition seeks to avoid classifying acquired firms as exiting due to financial distress simply because their stocks cease to trade. Using this criteria, we calculate a pre-exercise firm exit rate of $Pr^{\text{pre}}(exit) \approx 1.1\%$ per year, while $Pr^{\text{post}}(exit)$ is roughly 1/3 that rate.

Let N^{pre} and N^{post} be the numbers of pre-exercise and post-exercise firms. Let q be the *birth rate* of pre-exercise firms. To estimate the rate of option exercise, we note that our steady state assumptions imply

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{N^{\text{pre}}}{N^{\text{post}}} = 0 = \frac{N^{\text{pre}}}{N^{\text{post}}}\left(\frac{1}{N^{\text{pre}}}\frac{\mathrm{d}N^{\text{pre}}}{\mathrm{d}t} - \frac{1}{N^{\text{post}}}\frac{\mathrm{d}N^{\text{post}}}{\mathrm{d}t}\right).$$

The inflow and outflow of firms from the population must satisfy:

$$\frac{\mathrm{d}N^{\text{pre}}}{\mathrm{d}t} = (q - Pr^{\text{pre}}(exit) - Pr^{\text{pre}}(exercise))N^{\text{pre}} \tag{33}$$

$$\frac{\mathrm{d}N^{\text{post}}}{\mathrm{d}t} = Pr^{\text{pre}}(exercise)N^{\text{pre}} - Pr^{\text{post}}(exit)N^{\text{post}} \tag{34}$$

$$g = \frac{\frac{\mathrm{d}N^{\text{pre}}}{\mathrm{d}t} + \xi\frac{\mathrm{d}N^{\text{post}}}{\mathrm{d}t}}{N^{\text{pre}} + \xi N^{\text{post}}}. \tag{35}$$

The last equation is the total rate of growth of assets across all firms. One can solve these equations to derive

$$Pr^{\text{pre}}(exercise) = (g + Pr^{\text{post}}(exit))\frac{1-n}{n}.$$

We calculate g each December as $g=(totasset_t/totasset_{t-1})-1$ using 1980 to 2004 data.[18] The median value for g is 9.94% per year and the distribution is right-skewed. Note, the above equation is sensitive to estimates of n when n is close to 0.90. For example,

[18] Using longer time series is problematic. There is a sudden increase in quarterly Compustat listings in 1973 and large increases in the asset base in 1965 and 1971.

$Pr^{\text{pre}}(exercise)$ more than doubles when $\hat{n}$ decreases from 0.9 to 0.8. We therefore take $Pr^{\text{pre}}(exercise) \in [0.01, 0.03]$ to be a reasonable range. We are able to more easily calibrate our model to the target moments in Table 3 when $Pr^{\text{pre}}(exercise)$ is in the upper part of this range.

In reality it is very difficult to distinguish between actual firms that do or do not possess valuable growth options. One drawback of using physical size to distinguish between pre-exercise and post-exercise firms is that it presumes that the largest firms do not have growth options. While this is clearly not the case, it may be a reasonable first-order approximation to assume large firms have significantly fewer growth options.

We end by calculating the probability of exit due to distress in our model. For the post-exercise firms, we calculate the probability that a given firm with $p_t \in [\underline{p}^{\text{post}}, \infty)$ today will have $p_t < \underline{p}^{\text{post}}$ one quarter from today:

$$Pr(p_{t+0.25} \leqslant \underline{p}^{\text{post}} | p_t) = N\left(\frac{\underline{p}^{\text{post}} - e^{-\theta/4} p_t - (1 - e^{-\theta/4})\mu/\theta}{\sigma\sqrt{(1 - e^{-\theta/2})/2\theta}}\right), \tag{36}$$

where $N(\cdot)$ is the standard normal cumulative distribution function. We assume that the steady state distribution of firms mimics that of the unconditional log revenue price in the interval $[\underline{p}^{\text{post}}, \infty)$. The probability of exit due to distress is now calculated using the assumed steady state distribution of firms. Specifically, the post-firm's quarterly probability of exit is

$$Pr^{\text{post}}(exit, .25) \equiv \frac{\int_{\underline{p}^{\text{post}}}^{\infty} Pr(p_{t+0.25} \leqslant \underline{p} | p) \pi(p) \, \mathrm{d}p}{\int_{\underline{p}^{\text{post}}}^{\infty} \pi(p) \, \mathrm{d}p}, \tag{37}$$

where $\pi(p)$, the probability density for the unconditional distribution of p, coincides with a normal density with mean μ/θ and variance $(\sigma)^2/2\theta$. A similar expression is calculated for the pre-exercise firms with the important exception that now $p_t \in [\underline{p}^{\text{pre}}, \bar{p}]$. Since $Pr^{\text{post}}(exit)$ is the total annual exit rate for the post-exercise firms, λ^{post} can be calculated as $\lambda^{\text{post}} = Pr^{\text{post}}(exit) - 4Pr^{\text{post}}(exit, .25)$. A similar expression describes λ^{pre}. Our parameterizations are constrained by the requirement that λ^{pre} and λ^{post} are nonnegative.

Appendix D. Notes on numerical analysis of momentum strategies

Introduction

We create a population of model firms by choosing a limited number of parameter combinations, including both pre-exercise and post-exercise firms, and choosing different initial values of the log price of each firm's output good. Our methodology is akin to discretizing an assumed steady state distribution of firms on a grid rather than simulating firms. Our methodology allows us to efficiently calculate cross-sectional and time-series properties. In total, our population consists of $7 \times 2 \times 10 \times 2 \times 3 = 840$ different firms. Below, we describe how we create each firm in the population.

Seven parameter combinations

Our population of model firms consists of seven different parameterizations indexed by i. We start with the base-case parameter values shown in Table 1. We vary only the observable variables relating to the volatility of revenues (σ) and costs (K). The seven combinations of these two variables are shown in Table 3, Panel A.

Pre-exercise and post-exercise firms

We consider pre-exercise and post-exercise versions of each parameterization. The pre-versus post-status is indexed by k.

Ten initial log price values

We consider ten different initial log price values for each firm. The different values are indexed by j. A model firm is associated with a set parameters θ^i, σ^i, μ^i, and μ^{*i} and therefore a specific log price process defined by Eq. (5). The unconditional true distribution (i.e., not risk-adjusted) of this process is normally distributed with mean μ^i/θ^i and variance $(\sigma^i)^2/2\theta^i$. Let $N(\cdot)$ be the standard normal cumulative distribution function. Then $Pr^i(\underline{p}^{\text{pre},i}) = N\left(\frac{\underline{p}^{\text{pre},i}-\mu^i/\theta^i}{\sigma^i/\sqrt{2\theta^i}}\right)$ is the unconditional probability that the log price process (p_t^i) is below $\underline{p}^{\text{pre},i}$, and $Pr^i(\bar{p}^i) = N\left(\frac{\bar{p}^i-\mu^i/\theta^i}{\sigma^i/\sqrt{2\theta^i}}\right)$ is the unconditional probability that the log price is below$\bar{p}^i$.

We assume our pre-exercise model firms are evenly situated in the probability interval, $[Pr^i(\underline{p}^{\text{pre},i}), Pr^i(\bar{p}^i)]$. Specifically, the jth pre-exercise firm is at the “probability point” $Pr^i(\underline{p}^{\text{pre},i}) + (2j-1)/20(Pr^i(\bar{p}^i) - Pr^i(\underline{p}^{\text{pre},i}))$. Translating this back into log prices, the ten pre-exercise firms are distinguished by the following initial values of log revenues:

$$p_0^{ij,k=pre} = \frac{\mu^i}{\theta^i} + \frac{\sigma^i}{\sqrt{2\theta^i}} N^{-1}\left(Pr^i(\underline{p}^{\text{pre},i}) + \frac{2j-1}{20}(Pr^i(\bar{p}^i) - Pr^i(\underline{p}^{\text{pre},i}))\right), \quad j = 1, \ldots, 10. \tag{38}$$

Similarly, we assume that the ten post-exercise firms are evenly situated in the probability interval, $[Pr^i(\underline{p}^{\text{post},i}), 1]$, where $Pr^i(\underline{p}^{\text{post},i}) = N(\frac{\underline{p}^{\text{post},i}-\mu^i/\theta^i}{\sigma^i/\sqrt{2\theta^i}})$ is the unconditional probability that the log price process is below $\underline{p}^{\text{post},i}$. The ten post-exercise firms are distinguished by the following initial values of log revenues:

$$p_0^{ij,k=\text{post}} = \frac{\mu^i}{\theta^i} + \frac{\sigma^i}{\sqrt{2\theta^i}} N^{-1}\left(Pr^i(\underline{p}^{\text{post},i}) + \frac{2j-1}{20}(1 - Pr^i(\underline{p}^{\text{post},i}))\right), \quad j = 1, \ldots, 10. \tag{39}$$

We emphasize that although the twenty firms just described share a similar price process, every individual firm is assumed to be distinct in the sense that the only correlations in the revenues of any two firms is due to the common variation with the pricing kernel (i.e., due to ρ^i).

Two systematic shocks and three idiosyncratic shocks

Over the next Δt time interval, a proportion $(\lambda^{i,k}\Delta t)$ of firms of type *ik* receive a Poisson shock causing them to exit. Each firm that does not exit receives a shock to its revenues:

$$p_1^{ijk} = p_0^{ijk} e^{-\theta^i \Delta t} + \frac{\mu^i}{\theta^i}(1 - \mathrm{e}^{-\theta^i \Delta t}) + \sigma^i \sqrt{\frac{1 - e^{-2\theta^i \Delta t}}{2\theta^i}} \widetilde{x}_1 \sqrt{\Delta t},$$

where the shock, $\widetilde{x}_1 = \rho^i \widetilde{\varepsilon}_m + \sqrt{1-(\rho^i)^2}\widetilde{\zeta}$, is composed of a systematic $(\widetilde{\varepsilon}_m)$ and an idiosyncratic $(\widetilde{\zeta})$ component; $\widetilde{\varepsilon}_m$ is assumed to be binomial with an equal likelihood of being $\{-1, +1\}$, and $\widetilde{\zeta}$ is trinomial with an equal likelihood of being $\{-\sqrt{\frac{3}{2}}, 0, +\sqrt{\frac{3}{2}}\}$. It's easy to check that $\widetilde{x}_1$ has mean zero and unit variance. As $\varDelta \to 0$, the evolution of p_1^{ijk} approaches that of the diffusion in Eq. (5). For a firm that does not exit due to a Poisson shock and that experiences the shock $\widetilde{x}_1$, we calculate the return over the first quarter as

$$R_1^{ijk} = \exp((\beta^{ik}(p_0^{ijk})SR\rho^i\sigma^i + r)\Delta t)e^{\lambda^{ik}\Delta t} \frac{\exp(\beta^{ik}(p_0^{ijk})\sigma^i \widetilde{x}_1 \sqrt{\Delta t})}{\mathrm{E}[\exp(\beta^{ik}(p_0^{ijk})\sigma^i \widetilde{x}_1 \sqrt{\Delta t})]}. \quad (40)$$

The term $(\beta^{ik}(p_0^{ijk})SR\rho^i\sigma^i + r)\Delta t$ is the expected log return for a firm characterized by the indices *ijk*, where the firm's beta is $\beta^{ik}(p_0^{ijk})$ at the initial log price p_0^{ijk} (see Eq. (3)), while $SR\rho^i\sigma^i$ is the risk premium. For quarterly returns we set $\Delta t = \frac{1}{4}$. Firms that survive the Poisson shock have a higher return than expected, by $e^{\lambda^{ik}\Delta t}$. The term $\beta^{ik}(p_0^{ijk})\sigma^i \widetilde{x}_1 \sqrt{\Delta t}$ is the shock to returns. The normalization by the expected value of $\exp(\beta^{ik}(p_0^{ijk})\sigma^i \widetilde{x}_1 \sqrt{\Delta t})$ ensures the expected returns correspond to the first term and also introduces a Jensen's convexity term to the realized returns. If, upon receiving a negative realization of $\widetilde{x}_1$, a firm loses 90% or more of its value, we consider it removed from the population.

The total number of possible one-quarter realizations of returns corresponds to $(7 \times 2 \times 10$ types of firms$) \times (2$ systematic shock realizations$) \times (3$ idiosyncratic shock realizations$)$ for a total of 840 one-period firm returns. We now have a population of 840 firms that are heterogeneous in revenue volatility, costs, market value, market-to-book, and past one-quarter returns. We condition (i.e., sort) on systematic outcomes, a firm's previous one-quarter returns, and firm attributes, and then calculate expected returns for the next quarter. Since pre–exercise firms comprise 90% of all firms (see Appendix C), the returns of pre-exercise firms are assumed to appear nine times more frequently than the returns of post-exercise firms. When we sort or calculate value-weighted returns, we take into account the relative numbers of pre-exercise and post-exercise firms. A firm's *expected return* in the second quarter depends on the price reached at the end of the first quarter (p_1^{ijk}). If a firm does not exercise its growth option, its expected return is calculated as

$$\mathrm{E}_1[\widetilde{R}_2^{ijk}] = \exp((\beta^{ik}(p_1^{ijk})SR\rho^i\sigma^i + r)\Delta t). \quad (41)$$

If a pre-exercise firm with initial log revenue $p_0^{ij,k=\mathrm{pre}}$ receives a shock that takes it past the exercise threshold, then to avoid discontinuities resulting from our discretization, we treat it as a "portfolio" of two firms, where one is a pre-exercise firm with value $V^{\mathrm{pre},i}(\bar{p}^i)$ and the other is a post-exercise firm with value $\kappa V^{\mathrm{post},i}(p_1^{ij,\mathrm{pre}})$, and $\kappa = \frac{V^{\mathrm{pre},i}(\bar{p}^i)}{V^{\mathrm{post},i}(\bar{p}^i)}$ is a dilution factor (due to the issuance of equity to finance the expansion). The pre-exercise firm receives the weight $w = (\bar{p}^i - p_0^{ij,\text{pre}})/(p_1^{ij,\text{pre}} - p_0^{ij,\text{pre}})$. The post-exercise firm receives the weight $1 - w$. Thus, the post-exercise firm is more prominent in this ‘‘portfolio’’ of firms if the initial log price is very close to the exercise threshold. The expected returns are calculated by value-weighting the returns from (41), which reflects size differences between pre- and post-exercise firms in a given portfolio. We do this for up and down markets separately (since these would not appear simultaneously in the time series) and then average the results.

References

Asness, C., 1997. The Interaction of value and momentum strategies. Financial Analysts Journal March/April, 29–36.

Banerjee, S., Kaniel, R., Kremer, I., 2006. Momentum as an outcome of differences in higher order beliefs. Working paper, Stanford University.

Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment. Journal of Financial Economics 49, 307–343.

Baron-Adesi, G., Whaley, R.E., 1987. Efficient analytic approximation of American option values. Journal of Finance 42, 301–320.

Berk, J.B., Green, R.C., Naik, V., 1999. Optimal investment, growth options, and security returns. Journal of Finance 54, 1553–1607.

Bernard, V.L., Thomas, J.K., 1989. Post-earnings announcement drift: delayed price response or risk premium? Journal of Accounting Research 27, 1–36.

Bhojraj, S., Swaminathan, B., 2004. Macromomentum: returns predictability in international equity indices. Journal of Business 79, 429–451.

Biais, B., Bossaerts, P., Spatt, C., 2005. Equilibrium asset pricing and portfolio choice under asymmetric information. Working paper, Caltech.

Brekke, K.A., Øksendal, B., 1991. The high contact principle as a sufficiency condition for optimal stopping. In: Lund, D., Øksendal, B. (Eds.), Stochastic models and option values. North-Holland, Amsterdam, pp. 187–208.

Brennan, M.J., Schwartz, E.S., 1984. Optimal financial policy and firm valuation. Journal of Finance 39, 593–607.

Brennan, M.J., Schwartz, E.S., 1985. Evaluating natural resource investments. Journal of Business 58, 133–155.

Campbell, J.Y., 2003. Consumption based asset pricing. in: Constantinides, G., Harris, M., Stulz, R., (Eds.), Handbook of the Economics of Finance, vol. IB. North-Holland, Amsterdam, pp. 803–887.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics: implications for the cross-section of returns. Journal of Finance 59, 2577–2603.

Chan, L.K.C., Jegadeesh, N., Lakonishok, J., 1996. Momentum strategies. Journal of Finance 51, 1681–1713.

Chan, L.K.C., Jegadeesh, N., Lakonishok, N., 1999. The profitability of momentum strategies. Financial Analysts Journal 55, 80–90.

Chordia, T., Shivakumar, L., 2002. Momentum, business cycle, and time-varying expected returns. Journal of Finance 57, 985–1019.

Conrad, J., Kaul, G., 1998. An anatomy of trading strategies. Review of Financial Studies 11, 489–519.

Cooper, M., Gutierrez Jr., R.C., Hameed, A., 2004. Market states and momentum. Journal of Finance 59, 1345–1365.

Daniel, K., Titman, S., 1999. Market efficiency in an irrational world. Financial Analysts Journal 55, 28–40.

Daniel, K., Titman, S., 2006. Market reactions to tangible and intangible information. Journal of Finance 61, 1605–1643.

Daniel, K., Hirshleifer, D., Subrahmanyam, A., 1998. Investor psychology and security market under- and overreactions. Journal of Finance 53, 1839–1885.

Dixit, A., 1989. Entry and exit decisions under uncertainty. Journal of Political Economy 97, 620–638.

Duffie, D., Zame, W., 1989. The Consumption based capital asset pricing model. Econometrica 57, 1279–1297.

Dumas, B., 1991. Super contact and related optimality conditions. Journal of Economic Dynamics and Control 15, 675–685.

Fama, E.F., French, K.R., 1992. The cross section of expected stock returns. Journal of Finance 47, 427–465.
Fischer, E.O., Heinkel, R., Zechner, J., 1989. Dynamic capital structure choice: theory and tests. Journal of Finance 44, 19–39.
George, T.J., Huang, C., 2004. The 52-week high and momentum investing. Journal of Finance 59, 2145–2176.
Gomes, J., Kogan, L., Zhang, L., 2003. Equilibrium cross-section of returns. Journal of Political Economy 111, 693–732.
Griffin, J.M., Ji, S., Martin, J.S., 2003. Momentum investing and business cycle risk: evidence from pole to pole. Journal of Finance 58, 1547–2515.
Grinblatt, M., Han, B., 2005. Prospect theory, mental accounting, and momentum. Journal of Financial Economics 78, 311–339.
Grundy, B.D., Martin, J.S., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. Review of Financial Studies 14, 29–78.
Hong, H., Stein, J.C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. Journal of Finance 54, 2143–2184.
Hong, H., Lim, T., Stein, J.C., 2000. Bad news travels slowly: size, analyst coverage, and the profitability of momentum strategies. Journal of Finance 55, 265–295.
Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. Journal of Finance 48, 65–91.
Jegadeesh, N., Titman, S., 1995. Overreaction, delayed reaction, and contrarian profits. Review of Financial Studies 8, 973–993.
Jegadeesh, N., Titman, S., 2001a. Profitability of momentum strategies: an evaluation of alternative explanations. Journal of Finance 56, 699–720.
Jegadeesh, N., Titman, S., 2001b. Momentum. Working paper, University of Illinois.
Jegadeesh, N., Titman, S., 2002. Cross-sectional and time-series determinants of momentum returns. Review of Financial Studies 15, 143–157.
Jiang, G., Lee, C.M.C., Zhang, Y., 2005. Information uncertainty and expected returns. Review of Accounting Studies 10, 185–221.
Johnson, T.C., 2002. Rational momentum effects. Journal of Finance 57, 585–608.
Lakonishok, J., Shleifer, A., Vishny, R.W., 1994. Contrarian investment, extrapolation and risk. Journal of Finance 49, 1541–1578.
Leahy, J.V., 1993. Investment in competitive equilibrium: the optimality of myopic behavior. Quarterly Journal of Economics 108, 1105–1133.
Lee, C.M.C., Swaminathan, B., 2000. Price momentum and trading volume. Journal of Finance 55, 2017–2069.
Leland, H.E., 1994. Corporate Debt Value, Bond covenants and optimal capital structure. Journal of Finance 49, 987–1019.
Lewellen, J., 2002. Momentum and autocorrelation in stock returns. Review of Financial Studies 15, 533–563.
Liu, X., Warner, J., Zhang, L., 2004. Economic fundamentals, risk and momentum profits. Working Paper, University of Rochester.
Moskowitz, T.J., Grinblatt, M., 1999. Do industries explain momentum? Journal of Finance 54, 1249–1290.
Nagel, S., 2001. Is it overreaction? The performance of value and momentum strategies at long horizons. Working paper, London Business School.
Novy-Marx, R., 2006. Investment cash flow sensitivity and the value premium. Working paper, University of Chicago.
Ross, S.A., 1976. The arbitrage theory of capital asset pricing. Journal of Economic Theory 13, 341–360.
Rouwenhorst, K.G., 1998. International momentum strategies. Journal of Finance 53, 267–284.
Rouwenhorst, K.G., 1999. Local return factors and turnover in emerging stock markets. Journal of Finance 54, 1439–1464.
Rubinstein, M., 1983. Displaced diffusion option pricing. Journal of Finance 38, 213–217.
Sadka, R., 2004. Momentum and post-earnings-announcement drift anomalies: the role of liquidity risk. forthcoming. Journal of Financial Economics 80, 309–349.
Strobl, G., 2006. Time-varying information asymmetry and the disposition effect. Working paper, University of North Carolina at Chapel Hill.
Vuolteenaho, T., 2002. What drives firm-level stock returns? Journal of Finance 57, 233–264.
Wang, K.Q., 2005. Beta and momentum. Working paper, University of Toronto.
Zhang, L., 2005. The value premium. Journal of Finance 60, 67–103.

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