Merrill Lynch

CREDIT DERIVATIVES

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Derivatives

Correlation Trading

A New Asset Class Emerges

Global

Credit Correlation Trading – The Next Wave

User-Friendly Investment Products, But Complex Trader Risks

Growing liquidity of CDS together with advances in analytical and risk management technology are enabling the structuring of ever more sophisticated products around investor needs. The single-tranche CDO, for example, has offered investors a user-friendly spread product combining attractive yield, desired rating and significant investment control. Dealers, however, are left with complex risks to manage. With the success of such products in the market, new risks such as correlation are now emerging as distinct asset classes.

Standardised Tranches & Implied Correlation

The growth of CDS indices is also proving to be a big factor. These indices provide established portfolios upon which standardised tranches can be structured. Moreover, the trading prices of these tranches provide **a market view of correlation** (or implied correlation). This helps dealers to better price bespoke tranches but it also enables investors and traders to participate in a more transparent and liquid correlation market.

User Guide to Correlation Trading

This report provides a user guide to correlation trading and covers topics including the following:

- Pricing correlation products - what is a copula?
- Delta hedging - managing first-order spread risk.
- Correlation sensitivity of different tranches.
- Spread convexity & Gamma - second-order spread risk.
- Instantaneous default risk.
- Correlation strategies.

Correlation Strategies

Correlation products can provide sophisticated tools to express market views. For example, we show how they can be used to express views on correlated market movements and spread convexity as well as to take a "cheap" short on an index.

Refer to important disclosures on page 38.
Analyst Certification on page 37.

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CONTENTS

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Back to Table of Contents

1. Correlation: Investing vs. Trading

Growth of Correlation Products

Credit correlation market is showing strong growth . . .

The market for structured credit correlation products continues to grow and is driven by a number of factors including the following:

- Investors' desire for additional yield;
- Dealers' need to manage new risks such as correlation and higher-order risks such as spread convexity in addition to managing first-order risks such as spread risk;
- The increasing liquidity of the underlying hedging instruments, i.e. single-name CDS, as well as the development of CDS index products such as iBoxx;
- The improvement in analytics that enable dealers to manage higher-order risks more efficiently.

Credit correlation products refer to portfolio-based products where the price of the product is a function of default correlation between the individual credits in the portfolio. These include products such as synthetic CDOs (including single-tranche), Nth-to-default (NTD) baskets and hybrids such as FX or IR structures linked to a CDO or NTD.

. . . driven by growth in bespoke single-tranche synthetic CDOs . . .

Growth has been particularly strong for **bespoke single-tranche** synthetic CDOs which provide an alternative means of enhancing yield without having to move down the ratings curve. The disappearing arbitrage in full capital structure synthetic CDO deals has also favoured the issuance of single tranches. Their additional attraction is that they are bilateral arrangements between the dealer and the investor and can be executed faster and cheaper than full capital structure synthetic CDOs. More importantly the single-tranche portfolio is typically driven around the investor's need where the investor can maintain a degree of control throughout the transaction. This differs from a full capital structure synthetic CDO where the portfolio is typically determined (and controlled) by the first loss (or equity) investor. The growth of bespoke tranches has also been facilitated by the dealer's ability to price and hedge them with single-name default swaps more effectively.

. . . and increasing liquidity of CDS indices and standardised tranches based on them

With the increasing liquidity in CDS indices such as iBoxx, **standardised tranches** based on the index portfolios are also beginning to gain popularity both in the US and Europe. The market is still relatively nascent but liquidity in standardised tranches is improving and providing dealers with a more efficient way to manage risk on their non-standardised (or bespoke) portfolios. Dealers with diversified correlation books, for example, can use CDS indices and their standardised tranches to provide relatively quick and cheap macro hedges which can later be refined and tailored according to the specific exposure. In addition, standardised tranches provide a **market view of correlation** at different points in the capital structure which enables better pricing of bespoke tranches in addition to creating a more transparent and liquid correlation market.

Investors vs. Traders

"Correlation Trading" refers to investments or trades that arise from long or short exposure to correlation products. Correlation trading is a generic term that applies to both outright correlation "investors" as well as the more relative-value-based correlation "traders".

Correlation "investors" are fundamental credit investors . . .

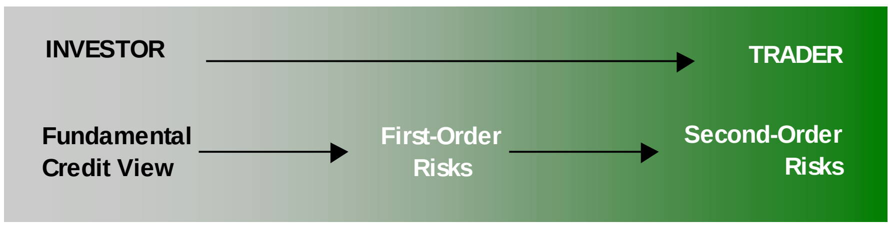
Correlation investors typically consist of fundamental credit investors who are looking to acquire cheap synthetic assets in an effort to enhance yield. These investors are **longer-term** players and typically invest in synthetic tranches or NTD baskets either in funded (CLN) or unfunded form. The tranche investor receives better yield relative to similar-rated single-name CDS or equivalent cash bond. However, in return for this additional yield, the investor gives up liquidity. Key investors include banks, insurance/reinsurance companies, monolines, middle market[1] and private clients.

. . . unlike the relative value correlation "traders"

Correlation traders on the other hand are mainly relative value players looking to acquire cheap convexity, volatility or correlation. Such traders typically combine different tranches, CDS indices and single name default swaps to isolate these risks and tailor their payoffs accordingly. They are typically **short-term** participants. Hedge funds, principal finance groups and dealers form the core group of correlation traders.

Correlation traders need to understand the risk components of correlation products in order to establish attractive relative value trades. They are concerned with managing not only first-order risks such as spread risk but also higher-order risks such as spread convexity. Correlation investors, however, typically hold outright positions to term that are subject to mark-to-market (MTM) risk with changes in spreads and implied correlation (though correlation neutral mezzanine tranches can also be created). Due to the long-term nature of the investment, the MTM risk would be mitigated if there is no default. However, for bespoke single-tranches with substitution rights, investors who are long these tranches need to understand how spread risk is managed[2] in order to understand the economics of substitution. Correlation investors, however, may well hedge "stressed" or deteriorating credits in the portfolio by buying protection on the full notional of that credit.

Chart 1: Risks Managed by Correlation Investors and Traders



Source: Merrill Lynch

We have covered the basics of single-tranche CDO's as a fundamental credit investment in an earlier report[3]. This report focuses on trading correlation products with particular focus on trading equity and senior tranches.

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[1] Middle market investors include smaller insurance companies, pension funds and second and third tier banks.
[2] Typically via Delta-hedging as described in a later section of this report.
[3] Refer to Merrill Lynch report titled *"Single-Tranche Synthetic CDOs"*, by Martin/Batchvarov/Kakodkar published on 20th August 2003.

Refer to important disclosures on page 38.

Back to Table of Contents

2. Default Correlation

Default correlation represents the likelihood of two (or more) credits defaulting together over a given time period. While it is a key driver of pricing correlation products it is also the most difficult to determine. Given that defaults are relatively rare events, the likelihood of simultaneous defaults is extremely low. Thus, historical default correlation is difficult to observe.

Estimating Default Correlation

Default correlation is difficult to estimate due to lack of historical data

Estimates of default correlation can be derived either by historical observation of proxies or by market implied measures. Empirical evidence suggests that default correlation is linked to credit rating and varies with time. In a negative economic environment, lower rated companies that are generally more leveraged are likely to default together and default correlation is expected to be higher. Conversely in a positive economic environment, default correlation is expected to be lower.

One method of estimating default correlation is from **structural models** (Merton type models). These models consider a firm in default when the value of its assets falls below a certain threshold amount, such as the face value of its debt. The probability of two firms defaulting is simply the probability that the market value of each firm's assets falls below their corresponding default threshold amounts. Stock market data is usually used to derive asset correlations as equity market information tends to be readily available and have a longer history from which to perform analysis. The asset correlation derived in this manner is deterministically related to the default correlation, i.e. one can be transformed into the other.

Market implied estimates are based on the increasing liquidity over the last few months of standardised tranches of index portfolios such as iBoxx. This development in addition to the adoption of a standardised framework to discuss correlation[4] has enabled market participants to compute implied default correlations from quoted bid-ask levels on standardised tranches of different seniority. These market-derived default correlation estimates can then be used as inputs to manage the risk of less liquid non-standardised (or bespoke) tranches with more certainty.

Spread Correlation vs. Default Correlation

Spread correlation serves as a useful proxy to understand P&L of correlation trades

Spread correlation and default correlation are different concepts and there does not seem to exist any obvious relationship between the two. However, one could argue that if two credit spreads are strongly correlated, it is likely that their defaults are also correlated. Spread correlation could be used as a reasonable proxy to default correlation given that credit spreads reflect market risk (including default risk) and are more frequently observable than defaults. However, there is a lack of high quality data for historical bond and CDS spreads to reliably compute spread correlation. Nonetheless, spread correlation is important for pricing correlation products because relative spread movements in a portfolio (or **realised correlation**) impacts the P&L of correlation trades. We discuss this in more detail later in this report.

[4] The Gausssian Copula model (outlined in the following section) is quickly becoming the standard framework to discuss default correlation.

Back to Table of Contents

3. Pricing Correlation Products

One of the fundamental problems in pricing portfolio products is the determination of a joint default probability distribution function for the multiple credits that form a typical portfolio. While the determination of a single-name survival curve is a relatively straightforward exercise, the process to derive a joint survival curve for multiple names in a portfolio is more complex as there is no explicit solution. In order to deal with this complexity, correlation dealers turned to "copula" functions to price correlation products.

Copula Functions

Copula functions are used to efficiently price correlation products

For large portfolios (typically greater than 50 credits), the number of joint default events becomes extremely large[5]. As a result the computation of joint default probabilities becomes more and more complex with an increase in the number of credits in the portfolio. The lack of an explicit solution to derive joint default probabilities requires the use of efficient numerical methods. **Copula functions** are numerical methods that **provide an efficient way to link multiple single-name (or unidimensional) survival curves to one multi-name (or multidimensional) survival curve**. Combined with an assumption for default correlation, the model can then be used to price an individual tranche. However, the large number of credits in the portfolio vastly increase the number of individual default correlation relationships within the portfolio increasing the complexity of the computation even further. One way to deal with this situation, for example, is to assume a constant default correlation for each tranche of a synthetic CDO.

Different types of copulas have been proposed in financial literature. These include the Gaussian, Student's t, Marshall-Olkin, Clayton, Frank, Gumbel, etc. Each one of these is useful to deal with different features of the individual underlying default probability distributions such as skewed distributions, extreme values, etc.

Gaussian Copula function is becoming the market standard . . .

In these early days of correlation trading, the **Gaussian Copula** model, due to its analytical tractability and small number of parameters required, is quickly becoming the **standard framework** that market participants are using to discuss default correlation. This is similar to the early days of options trading when the Black-Scholes model was used as a common framework to discuss volatility.

Copula functions are discussed in more detail in the Appendix at the end of this report.

Implied Default Correlation

. . . and is used to derive a market-implied default correlation

Under a Gaussian Copula model, the **premium** of an individual tranche is a function of the following:

- default correlation,
- underlying spreads in the portfolio,
- number of credits and relative size in portfolio,
- attachment point and width of the tranche,
- maturity of the deal,
- recovery rate on each credit, and
- risk-free discount curve.

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[5] For an N-credit portfolio, there are 2^N joint default events. For example, a 2 credit portfolio of X & Y has four joint default events: (1) Neither defaults; (2) X defaults; (3) Y defaults; (4) Both default.

Table 1: Implied Correlation for iBoxx CDX NA Tranches

	Implied Correlation	
Tranche	Bid	Offer
0-3%	24%	17%
3-7%	9%	23%
7-10%	17%	24%
10-15%	23%	32%
15-30%	28%	39%

iBoxx CDX NA mid = 52 bps; Maturity = 20 March 2009
Source: Merrill Lynch

Conversely, if we know the market premium for a single-tranche along with all the inputs (except default correlation) mentioned above, we can derive an **implied default correlation** from the model. This is similar to extracting an implied volatility from the Black Scholes model given the price of an option.

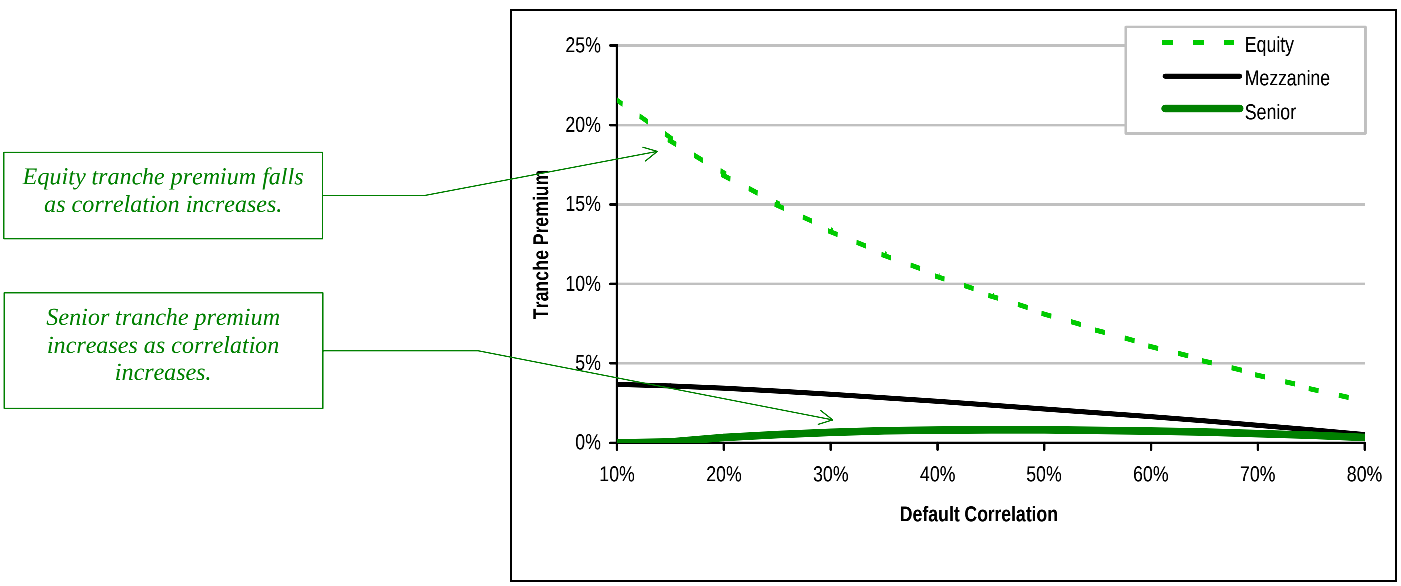
Table 1 highlights the bid/offer implied default correlations for different tranches of the iBoxx index. Implied correlations can be different for each tranche based on the demand and supply technicals. As a result, we observe a term structure of implied correlation as a function of subordination. This term structure can also be expected to change over time as the technicals change.

Tranche Premium vs. Default Correlation

The change in tranche premium with correlation is based on tranche subordination

As explained in an earlier publication on single-tranche CDOs[6], the tranche premium as a function of correlation depends on the tranche subordination. For all levels of correlation, the equity (first-loss) tranche premium is the highest reflecting the risk of first-loss and no subordination. The senior tranche is the least risky as it is protected by the subordination of the equity and mezzanine tranches. Chart 2 highlights tranche premium as a function of default correlation for a hypothetical portfolio.

Chart 2: Tranche Premia Versus Correlation



Source: Merrill Lynch

The expected loss of a portfolio is independent of correlation[7]. However, correlation determines how the expected losses are parcelled out across the capital structure. The portfolio loss distribution for different correlation assumptions is examined in detail in our publication referred to above. In summary:

Equity tranche is more risky at lower correlations . . .

At **low** default correlations, we expect the portfolio to have a high probability of a few defaults. As a result, the probability of loss at the senior tranche is low making it less risky. The probability of zero losses however is also small at low default correlation. The few defaults that occur, therefore, would be absorbed mainly by the equity tranche making it relatively more risky. The spread of the mezzanine tranche lies between the two.

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[6] See *"Single-Tranche Synthetic CDOs"* by Martin/Batchvarov/Kakodkar, 20th August 2003

[7] Expected loss is a function of probability of default and recovery rate.

. . . while senior tranche risk rises with higher correlation

At **high** default correlations, the credits in the portfolio behave like one asset that either all default together or do not default at all. The finite probability of high losses would make the senior tranche more risky. However, the large probability of no defaults lowers the risk on the equity tranche. The spread of the mezzanine tranche again lies between the two.

Long or Short Correlation?

Tranche exposure determines a view on correlation . . .

An investor's view on correlation via exposure to the equity or senior tranche is summarised in Table 2. We highlight the following:

An investor can be **long correlation** by being either **long the equity** tranche or **short the senior** tranche.

An investor can be **short correlation** by being either **short the equity** tranche or **long the senior** tranche.

The sensitivity of a mezzanine tranche to correlation is function of its attachment point and size as well as its underlying credit spreads and lies in between the equity and senior tranches.

Table 2: Tranche Exposure to Reflect Correlation View

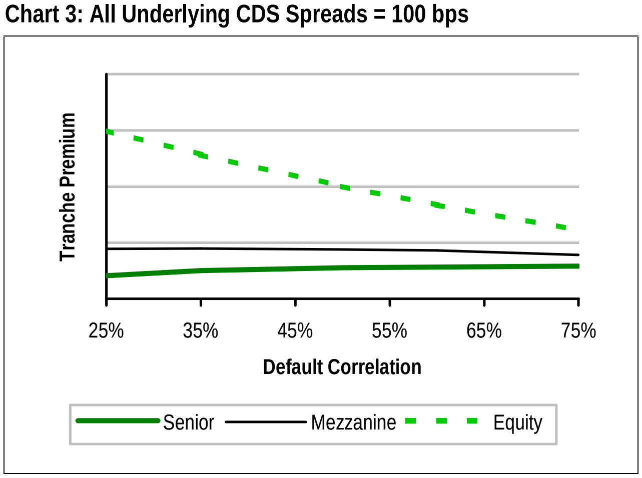
	Equity Tranche Exposure	Senior Tranche Exposure
Long Correlation	LONG	SHORT
Short Correlation	SHORT	LONG

Source: Merrill Lynch

. . . that can change over time

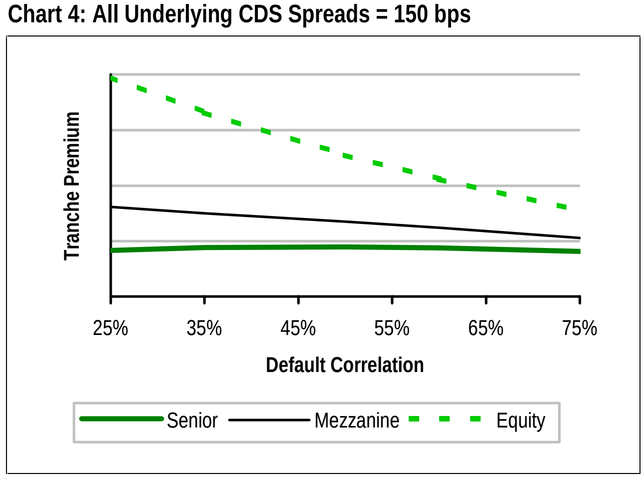
This sensitivity to correlation can change through time as a function of spreads, defaults and time decay. For instance, if all spreads in the underlying portfolio tighten significantly, a (correlation neutral) mezzanine tranche would begin to take on the correlation sensitivity of a senior tranche i.e. as correlation *increased*, the premium on the mezzanine tranche would *rise* and vice versa. Similarly, if all spreads widen significantly (see Chart 3 and Chart 4), the mezzanine tranche would begin to take on the correlation sensitivity of an equity tranche, i.e. as correlation increased, the premium on the mezzanine tranche would fall and vice-versa. However, the exact behaviour of a mezzanine tranche would be a function of the magnitude of the spread change and the attachment points of the mezzanine tranche. We have chosen suitable attachments points to demonstrate this behaviour in the charts below. We explore correlation sensitivity in more detail in a later section.

Chart 3: All Underlying CDS Spreads = 100 bps



Source: Merrill Lynch

Chart 4: All Underlying CDS Spreads = 150 bps



Source: Merrill Lynch

Back to Table of Contents

4. Delta Hedging

Dealers typically manage the spread risk of a tranche by using single-name CDS as an offsetting hedge. The amount of CDS protection bought or sold on credits in the CDO portfolio is defined by its Delta. The directionality and magnitude of Delta can be more difficult to understand however, and depends on many factors including subordination, spread, correlation and time.

Hedging Single-Tranche Positions

Managing The Risks of Correlation Products

Dealers need to manage the multiple risks in correlation products

An ‘ideal’ hedge for a dealer who has shorted (bought protection on) a single-tranche of a CDO would be to enter into an identical offsetting transaction. This would involve selling protection on an identical tranche of the same underlying portfolio. However, due to the bespoke nature of the single-tranche CDO market, the availability of such an offsetting transaction is very unlikely. As a result, a dealer will focus on **managing the spread risk of a single-tranche position** (first-order risk) via the underlying single-name CDS market, and apply a trading-book approach to managing other risks such as implied correlation etc.

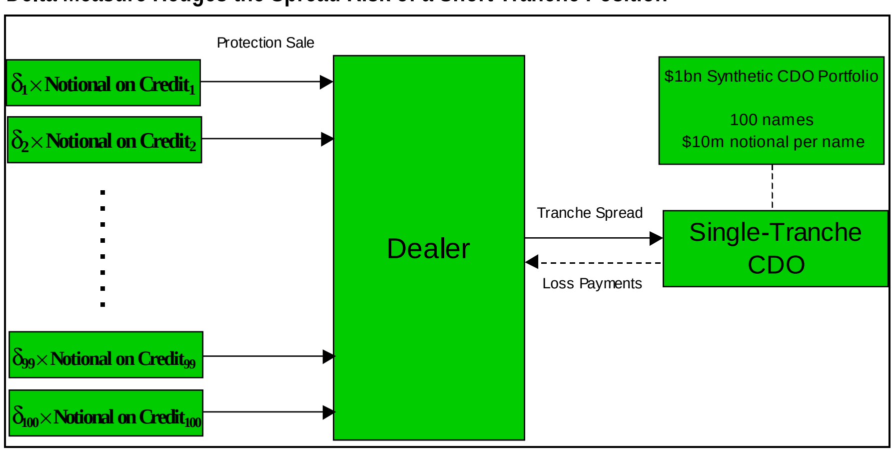
However, dealers can enter into offsetting transactions to hedge the risk of standardised tranches of CDS indices. Moreover, these standardised tranches can also be used as an approximate offsetting hedge for a bespoke tranche, given that there may well be a reasonable overlap in reference entities of both CDO pools. However, such a hedge would not be a perfect hedge.

Defining Delta

Delta hedging is used to manage spread risk

All single-tranche positions are subject to MTM movements as credit spreads on names in the underlying portfolio move about over the life of the transaction. To hedge the spread risk of a *short position* in a tranche, a dealer needs to *sell protection* on each of the underlying credits in the portfolio according to the **Delta** (δ) measure. Similarly, to hedge the spread risk of a *long position* in a tranche, a dealer needs to *buy protection* on each of the underlying names in the portfolio according to the Delta measure. In Chart 5, we show the hedging behaviour of a dealer who is short a single-tranche CDO.

Chart 5: Selling Protection on each name in the Underlying Portfolio According to the Delta Measure Hedges the Spread Risk of a Short Tranche Position



Source: Merrill Lynch

We define the Delta of a credit in the underlying portfolio as the amount of protection the dealer sells (buys) on that name to hedge the MTM risk of a short (long) tranche position due to movements in the credit spread of that name.

More formally, we can define the Delta of a credit as the ratio of the spread sensitivity of the tranche position to the spread sensitivity of that credit:

$$\text{Delta of Credit} = \frac{\text{Change in Mark-To-Market of Tranche}}{\text{Change in Mark-To-Market of Credit}}$$

This relationship defines the percentage of the notional amount of the credit that needs to be sold (bought) in order to hedge a short (long) tranche position.

For example, assume that a credit in the underlying portfolio, originally with a spread of 100bps, widens by 1bp. This spread widening translates into an absolute MTM change of €4,540 on the credit. If this spread widening also resulted in an absolute MTM change on the short tranche position of €2,107, then the Delta for that credit would be equal to **46.4**% (=2,107/4,540). This means that to hedge the MTM risk of the tranche to small moves in the credit spread of that name, the dealer would sell protection on 46.4% of the notional of that credit. If the credit had a notional value of €10m in the CDO pool, the amount of protection sold would be €4,640,000.

Deltas are calculated by "brute force"

There are no explicit formulas to calculate Delta. Instead, Deltas are generally calculated using "brute force" by shifting the credit spreads on each individual name in the CDO portfolio by a small amount (1-10bps), and then calculating the resultant Mark-to-Market change of the tranche. **Deltas range from 0%-100% for a credit in a CDO.** The 'Delta' of a single-name CDS is 100% and the Delta of a 0%-100% tranche of a CDO is 100%.

Note that we calculate Delta for small spread changes in the underlying credits. For larger spread changes, a delta-hedged tranche is not totally immune to spread movements. In other words, a delta-hedged CDO position is still subject to *Spread Convexity* for larger spread moves, and as a result the Deltas will need to be dynamically rebalanced throughout the life of the transaction.

Dynamic Hedging

Deltas need to be rebalanced to remain fully hedged for spread movements . . .

In the context of correlation products, dynamic hedging is the process of delta-hedging a single-tranche CDO over time. Dealers will typically manage the spread risk of a single-tranche position by dynamically rebalancing the Delta hedges as spreads move. However there are practical limits to such a process. First, there is a cost associated with rebalancing the hedges. Whilst underlying spread curves move on a daily basis, rebalancing will typically be on a less frequent basis as buying and selling small portions of credit risk can become relatively expensive when bid-offers are taken into account. The more frequent the rebalancing of Deltas, the more expensive the hedging cost. Tight bid-offer spreads in the single-name CDS market would help mitigate the cost of rebalancing.

Second, it may not always be possible to transact single-name hedges in the precise Delta amounts that are produced from analytical models. Third, CDO collateral portfolios are constructed to be diverse in nature so that more senior tranches obtain higher credit ratings. However, not every name referenced in the portfolio will have the same liquidity in the underlying single-name CDS market. This may cause a conflict between the need to rebalance hedges on a frequent basis and the availability of liquid CDS contracts for which to hedge.

Other Risks of Correlation Products

. . . but still exposed to other risks

Delta-hedging immunises the dealer against small movements in the credit spreads of names in the underlying portfolio. However, a delta-hedged tranche is not a completely risk-free position for a dealer as there are a number of other risk factors that need to be managed. These additional risks are implied correlation risk, risk of a sudden and unexpected default in the portfolio, time-decay, spread convexity and recovery rate risk. As a result, dealers are motivated to do trades that reduce these risk factors such as placing other tranches of an existing CDO with other investors or, where possible, entering into similar offsetting transactions.

Delta Sensitivities

Deltas are sensitive to several factors

There are many sensitivities of Delta that can at first appear less straightforward. Deltas of individual credits[8] depend on the following parameters and will change as these parameters change through the life of the transaction:

- **Attachment point (i.e. Subordination) and Width of the Tranche**
- **CDS Spread of Underlying Credits and Relative Spread Movements**
- **Time Remaining until Maturity**
- **Correlation with other Credits in the Portfolio**
- **Recovery Rate assumption of the Credits**

In order to understand the directionality and magnitude of Deltas we explore these sensitivities with two CDO examples.

We define a **Discrete CDO** example with underlying collateral pool and tranching as detailed in Table 3. We also define a **Continuous CDO** on the same collateral pool where each tranche has a 1% width and attachment points vary by 1% increments.

As the CDO collateral pool is homogenous with respect to spread, recovery and correlation assumptions, the Delta of each credit will be identical for any given tranche. However, as actual collateral pools tend to be much more heterogeneous with respect to credit spreads, Deltas will typically be different for each credit.

Table 3: Discrete CDO Portfolio and Tranching

Property	Quantity
Underlying Portfolio	
Number of credits	50
Notional Per Credit	€10m
Total Portfolio Size	€500m
CDS Spread on *each* credit	100 bps
Recovery Rate on *each* credit	35%
Default Correlation	25%
Tranching	
Equity	0%-4%
Mezzanine	4%-8%
Senior	8%-12%
Super Senior	12%-100%

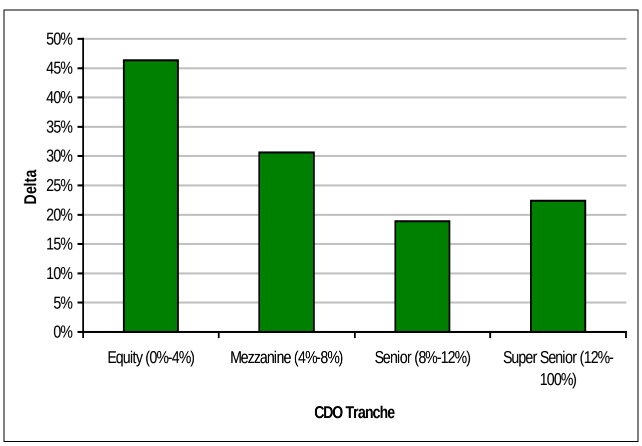
Source: Merrill Lynch

Delta as a Function of Subordination

Tranches lower in the capital structure are more risky as there is less subordination to act as a 'buffer' against defaults. As the equity (first-loss) tranche is the riskiest tranche in the capital structure, credits in the underlying portfolio have the highest equity-tranche Delta relative to the more senior tranches. For more senior tranches in the capital structure, the Delta of a credit is lower.

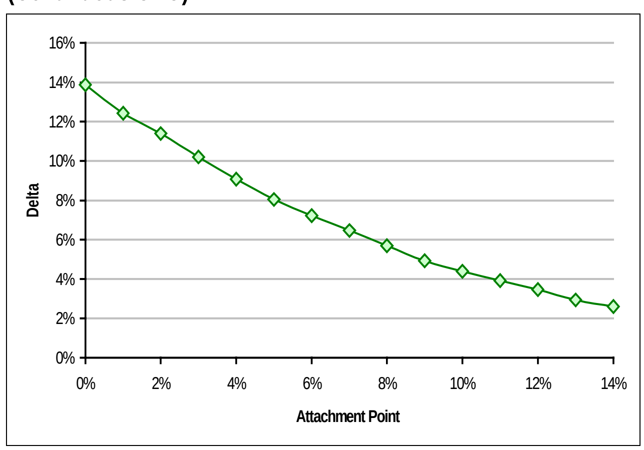
A credit's Delta is *higher* for a tranche with *less subordination*, and vice-versa.

Chart 6: Delta as a Function of Subordination (Discrete CDO)



Source: Merrill Lynch

Chart 7: Delta as a Function of Subordination (Continuous CDO)



Source: Merrill Lynch

[8] For an individual credit, the credit Delta is different for different tranches, i.e. the equity tranche Delta of a credit is different from its senior tranche Delta.

Deltas decrease with increase in subdordination . . .

As Delta can be thought of as representing the probability of losses in a tranche, tranches with less subordination have a higher Delta. Note however, that in the Discrete CDO example the super-senior tranche actually has a higher Delta than the senior tranche as the tranche width is significantly larger.
However, as a percentage of the notional size of the tranche ("leverage"), Delta for the super-senior tranche is lower than the Delta for the senior tranche. We talk more about the concept of Leverage at the end of this chapter.

Delta as a Function of Credit Spreads

. . . but depend on the tranche as a function of individual credit spreads

To show the dependence of Delta on individual credit spreads, we disperse the credit spreads in the underlying collateral pool of Table 3 such their average-spread remains at 1%. Spreads are adjusted to be linearly increasing in this instance.

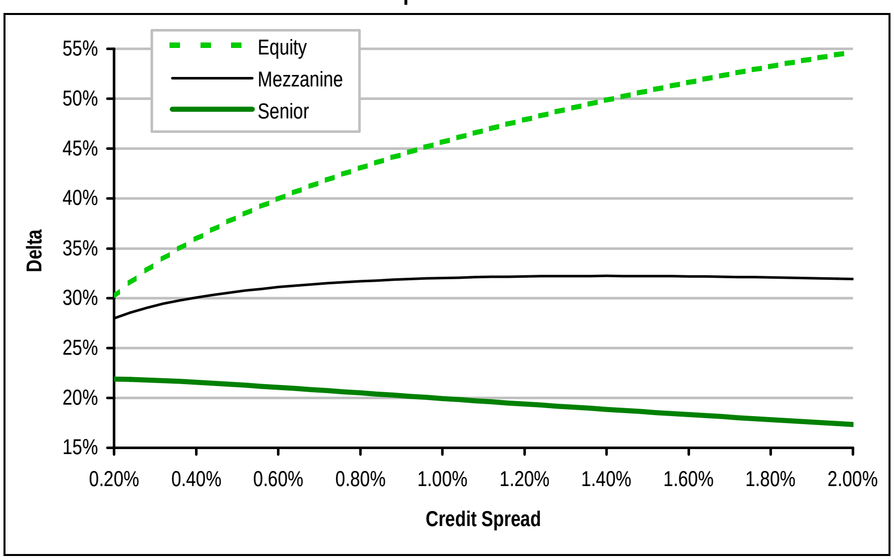
One way to conceptualise Delta is to think of ordering all of the credits in the underlying portfolio by their time to default. For constant recovery rate and correlation assumptions across all names, this will simply be a reflection of the credit spread of the name: **credits with a higher spread are expected to default before credits with a lower spread**. If a credit whose Delta we are calculating is towards the front of this 'queue' (i.e. higher spread) it is more likely to cause losses to the equity tranche and so its **equity tranche Delta will be higher**. If the credit is further back in this queue (i.e. lower spread) then its **equity tranche Delta will be lower**. Moreover, as credits further down the queue are likely to have a later default time, they are more likely to cause losses to the more senior tranches in the capital structure. As a result, the **senior tranche Delta of the credit will rise**.

For the equity tranche, a *higher* credit spread is associated with a *higher* Delta for that credit (relative to the average Delta of that tranche), and vice-versa.

For the senior tranche, a *higher* credit spread is associated with a *lower* Delta for that credit (relative to the average Delta of that tranche), and vice-versa.

For the mezzanine tranche, the Delta has less directionality with respect to credit spreads.

Chart 8: Deltas as a Function of Credit Spread



Assume Discrete CDO.
Source: Merrill Lynch

Refer to important disclosures on page 38.

This relationship is shown in Chart 8 where the equity tranche Deltas are higher for wider spread credits and the senior tranche Deltas are lower for wider spread credits. These results also apply to the directionality of Delta when credit spreads in the portfolio move throughout the life of the transaction:

For the equity tranche, Deltas on individual credits will *rise* as their credit spreads *widen* and vice-versa[9].

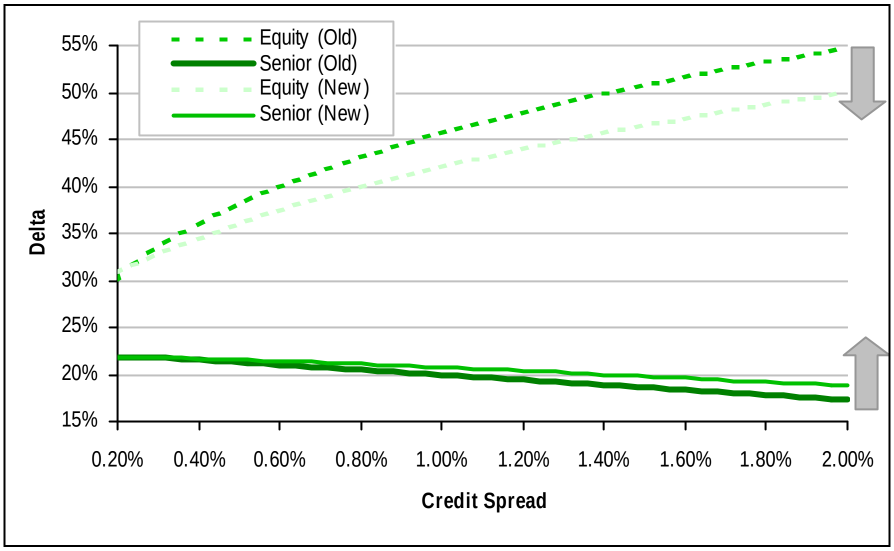
For the senior tranche, Deltas on individual credits will *fall* as their credit spreads *widen* and vice-versa.

■ Delta as a Function of Cumulative Spread Moves

Deltas move differently for different tranches as all spreads move together . . .

The **cumulative** spread movements of all the credits in the portfolio will also impact how the Deltas change. We analyse how Deltas move in this instance by shifting all credit spreads in the above example wider by 25bps. As can be seen in Chart 9 this has the impact of *decreasing* the equity tranche Deltas for *each* individual credit, but *increasing* the senior tranche Deltas for *each* individual credit. This is because shifting all credit spreads wider simultaneously has the impact of decreasing the probability of a small number of losses (lowering the relative risk of the equity tranche), whilst increasing the probability of a larger number of losses (increasing the relative risk of the senior tranche). The mezzanine tranche Deltas are less impacted.

Chart 9: Behaviour of Deltas as ALL Spreads in the underlying Portfolio are Shifted Wider



Assume Discrete CDO.
Source: Merrill Lynch

Similarly, when all credit spreads in the underlying portfolio are shifted tighter then the equity tranche Delta *increases* for *each* individual credit and the senior tranche Delta *decreases* for *each* individual credit. This is because shifting all credit spreads tighter simultaneously has the impact of increasing the probability of a small number of losses (increasing the risk of the equity tranche), whilst decreasing the probability of a larger number of losses (decreasing the risk of the senior tranche). The mezzanine tranche Deltas are less impacted.

---

[9] As the change in spread of any one credit impacts the weighted-average spread of the portfolio, there will be a small impact on the Deltas of the remaining credits. For example, shifting one credit from 100bps to 105bps increases its equity tranche Delta but the equity tranche Deltas of the other names in the portfolio fall marginally, and vice-versa.

We note however, that the **shape of the distribution of Deltas remains the same after cumulative spread movements**: For the equity tranche, wider spread credits still have a higher Delta than lower spread credits. For the senior tranche, wider spread credits still have a lower Delta than lower spread credits.

The equity tranche Deltas of *each* credit *decrease* as credit spreads across the entire portfolio widen and the equity tranche Deltas of *each* credit *increase* as credit spreads across the entire portfolio tighten.

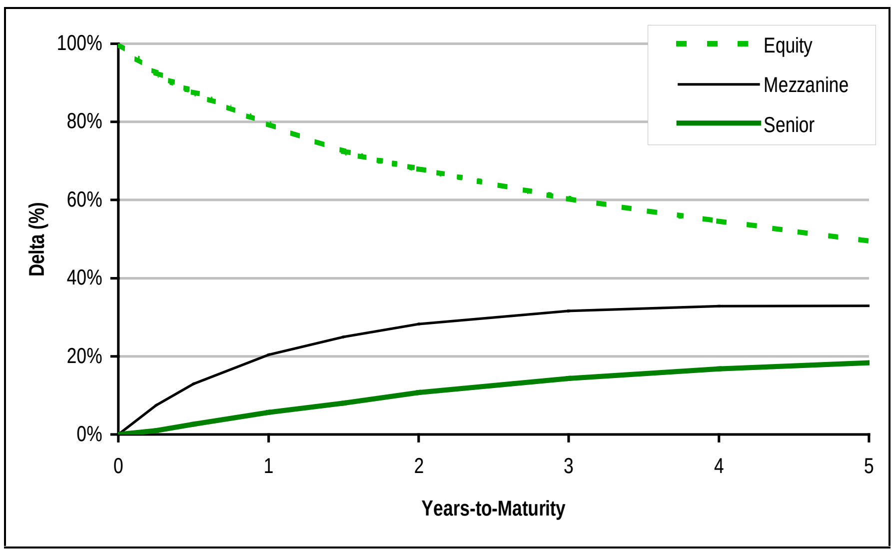
The senior tranche Deltas of each credit *increase* as credit spreads across the whole portfolio widen and the senior tranche Deltas of *each* credit *decrease* as credit spreads across the whole portfolio tighten.

■ Delta as a Function of Time

. . . or as time to maturity decreases . . .

Deltas will also change due to the passage of time even if credit spreads remain unchanged. Assuming no defaults in the underlying portfolio, the Delta of the equity tranche will *increase to 100%* as time to maturity approaches and accordingly, the Deltas of the Mezzanine and Senior tranches will *decrease to 0%*. This is because as maturity approaches, the more senior tranches become relatively less risky compared to the equity tranche as there is less time for defaults to accumulate such that the notionals of the mezzanine and senior tranches are reduced. The equity tranche becomes relatively more risky in the capital structure, and as a result its Delta tends to 100%.

Chart 10: Deltas over Time: Equity Tranche Delta tends to 100% as it Becomes Relatively Riskier



Assume Discrete CDO.
Source: Merrill Lynch

■ Delta as a Function of Correlation

. . . or correlation changes

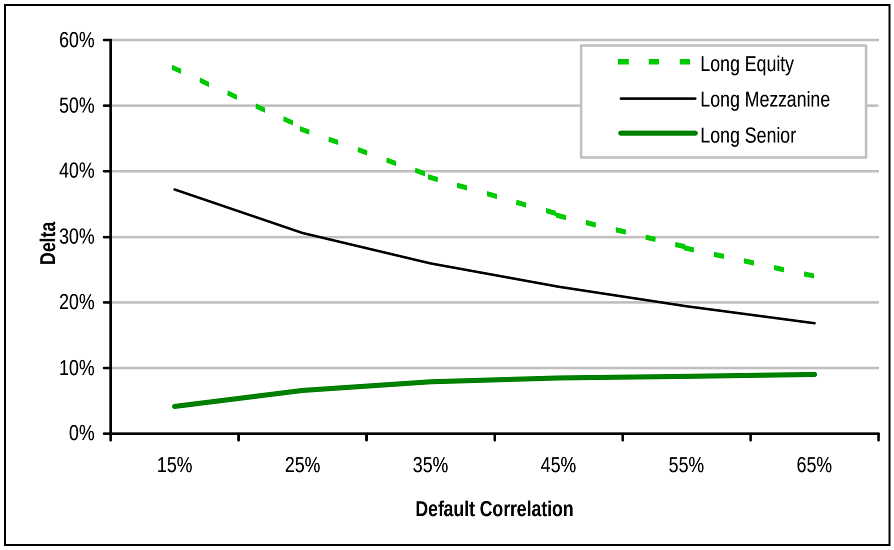
Deltas are also impacted by the changes in the underlying default correlation. If we increase the underlying default correlation assumption, we have the following conclusions:

- As correlation increases, the equity tranche Delta of a credit *decreases* and the senior tranche Delta *increases*.
- For every default correlation assumption, equity-tranche Deltas are higher than senior-tranche Deltas. Mezzanine-tranche Deltas remain in between.

As the correlation of the portfolio increases, more of the risk is parcelled to the senior tranche implying an increase in senior tranche Delta and a consequent decrease in equity tranche Delta.

Refer to important disclosures on page 38.

Chart 11: How Delta Changes as Default Correlation in the Portfolio Changes



Assume Discrete CDO. We have chosen a senior tranche with greater subordination to show the upward sloping nature of the senior curve.
Source: Merrill Lynch

Leverage "Lambda" of a Tranche

Leverage is a credit risk measure . . .

The **Leverage** (or "Lambda") of a tranche is an important spread risk measure. We define Leverage as equal to the **notional size of a tranche's Delta-hedge portfolio divided by the tranche's notional size**.

For example, in Table 3, each credit in the equity tranche has a Delta of 46.4%. As there are 50 names each of €10m in the CDO collateral pool, we have the following:

- The Delta-hedge portfolio has a notional size of €231.85mn (50*€10mn*46.4%).
- The equity tranche has a notional size of €20mn (=4%*€500mn).
- The Leverage of the equity tranche is 11.6x (=231.85/20).

Table 4: Calculating Leverage ("Lambda") for Discrete CDO Case

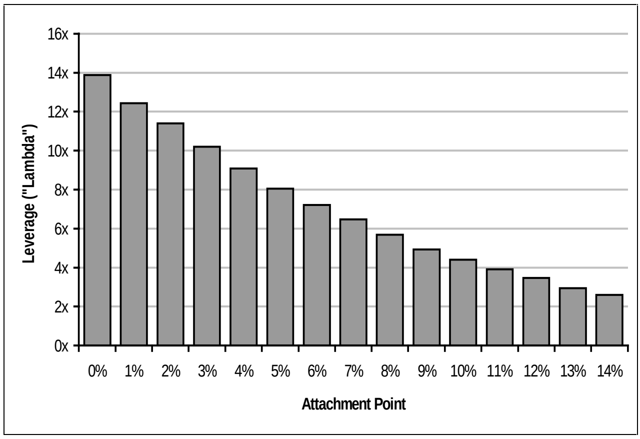
Tranche	Delta (%)	Notional Size of Delta-Hedge Portfolio (€)	Tranche Notional Size (€)	Leverage (Delta-Hedge Portfolio ÷ Tranche Notional)
Equity	46.4%	231,850,000	20,000,000	11.6x
Mezzanine	30.6%	153,100,000	20,000,000	7.7x
Senior	**18.9%**	94,420,020	20,000,000	4.7x
Super Senior	**22.4%**	111,800,000	440,000,000	0.3x

Source: Merrill Lynch

. . . that scales Delta by notional size of tranche

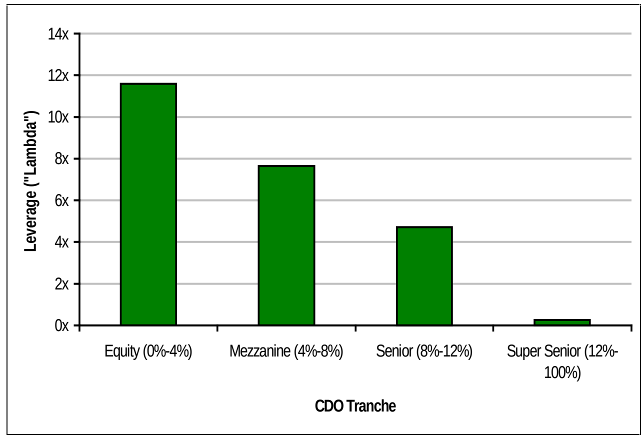
Effectively, Leverage 'scales' the Delta by the notional size of the tranche: the Delta of the super-senior tranche is actually larger than the Delta of the senior tranche, but its Leverage is smaller (Chart 13). Leverage gives an indication of how total risk is parcelled out between the different tranches – more leveraged tranches are those for which the spread risk is high in relation to the notional. Delta and Leverage are both good measures of the credit risk of a tranche.

Chart 12: Leverage ("Lambda") for a Continuous CDO



Source: Merrill Lynch

Chart 13: Leverage ("Lambda") for a Discrete CDO



Source: Merrill Lynch

Leverage measure used to delta-hedge standardised tranche via CDS index

Delta-Hedging a Standardised Tranche with a CDS Index

While underlying CDS will typically be used to delta-hedge a bespoke single-tranche position, standardised tranches of CDS indices can be delta-hedged via the underlying index itself. Suppose that a dealer buys protection on the 7%-10% tranche of iBoxx. The Delta-hedging calculations are as follows:

- 7%-10% tranche notional is $30mn as total portfolio size assumed is $1bn.
- Suppose that the iBoxx 7%-10% tranche has Leverage of 5x.
- Implied Delta portfolio is $150mn ($30mn*5).
- An iBoxx contract size of $25mn would imply a delta-hedge of 6 contracts.

Note that this hedge is an average delta-hedge. For a senior tranche where underlying spreads are dispersed as in iBoxx, the tighter names (with higher than average Deltas) would be underhedged whereas the wider names (with lower than average Deltas) would be overhedged. The reverse would be true for hedging a equity tranche position with the index.

Delta is also important outside of correlation trading

Importance of Delta

Whilst Delta is of practical importance in managing the spread risk of a single-tranche, investors in bespoke CDO tranches may conclude that Delta is not so important. However, Delta plays an important role outside of pure correlation trading for two reasons:

➢ **Adding CDOs to a Portfolio of Bonds or Loans**

It is not immediately clear how the risk/reward profile of a portfolio of bonds or loans changes by adding a bespoke CDO tranche to the portfolio. However, given that the Delta is a credit risk equivalent metric, CDO tranches can be 'replaced' by their Delta-equivalent credit portfolios. This provides a basis for which aggregate exposure to any one credit can be summed-up over the portfolio.

➢ **Substitution Mechanics in Managed Synthetic Deals**

Deltas are contractually used in managed single-tranche deals to determine the economics of substitution. Substitutions are typically permitted for reasons of credit deterioration or improvement of names in the underlying portfolio. The loss or gain from substitution will be a function of several factors:

- The Difference Between the Spreads of the Outgoing and Incoming Credits
- The Delta of both the Outgoing and Incoming Credits
- Remaining Spread Duration of the Tranche
- Notional Amount of the Credits

Refer to important disclosures on page 38.

5. Correlation Sensitivity (Rho)

Back to Table of Contents

Equity tranche investments are typically long correlation whilst senior tranche investments are typically short correlation. Given certain attachment points along the capital structure, a correlation invariant mezzanine tranche can be constructed.

Correlation Sensitivity and Subordination

Tranche sensitivities to correlation can change with time

As mentioned previously, different tranches along the capital structure have different sensitivities to movements in correlation. Equity tranche investments are typically *long* correlation positions (tranche premium decreases as correlation increases), senior tranche investments are *short* correlation (tranche premium increases as correlation increases), and mezzanine tranche investments are fairly correlation invariant depending on the respective attachment points. Over time however, tranche sensitivities to correlation can change if, for instance, spreads in the underlying portfolio move meaningfully wider or tighter, or if defaults erode the subordination beneath a tranche.

We define Rho as the MTM change of a tranche for a 1% change in the default correlation used to price the tranche. From the perspective of an investor in the equity tranche and using the discrete CDO example in Table 3, if we adjust our initial correlation assumption of 25% to 26%, the Equity tranche MTM for the investor increases by about €229k i.e. **Rho is positive for the equity tranche**. However, the same adjustment in correlation for an investor in the senior tranche results in a negative MTM of about €-31k i.e. **Rho is negative for the senior tranche**.

Some of the factors impacting Rho include:

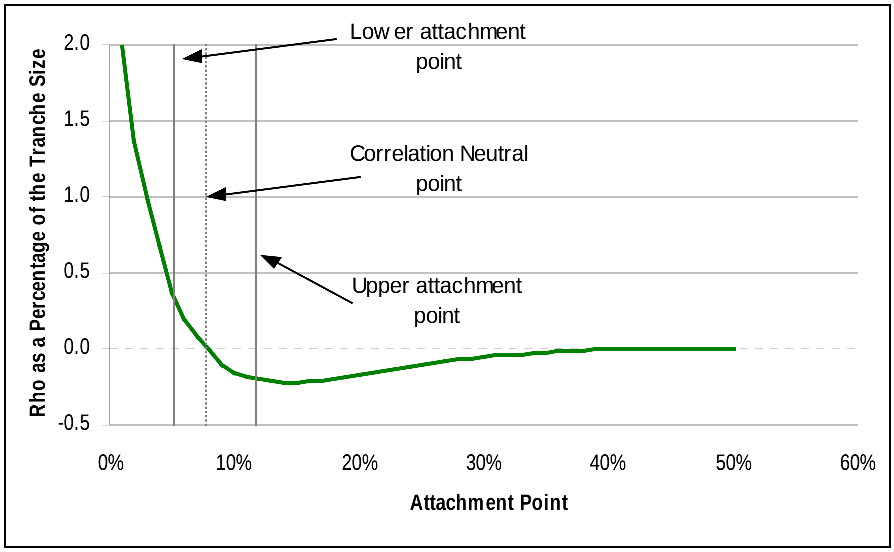
- Position in the Tranche i.e. Long or Short Tranche
- Tranche Subordination
- Average level of Credit Spreads in the underlying portfolio
- Time to Maturity of the Transaction
- Actual level of Correlation

Long Correlation positions (long equity or short senior) have a *positive* Rho.
Short Correlation positions (short equity or long senior) have a *negative* Rho.

In Chart 14 we show Rho as a function of subordination for the Continuous CDO example from the perspective of an investor who has long exposure. Rho is positive for the equity tranche and then tends to a negative value for more senior tranches. Note that Rho tends to zero for very high attachment points. In between the equity and senior tranches, there lies a C*orrelation Neutral Point*, where Rho is zero. We can define attachment points around this Correlation Neutral Point to construct a **Correlation Neutral Mezzanine Tranche**.

We note also that a CDS and a portfolio of CDS (such as a CDS index) has no sensitivity to correlation. As a result, a Delta-hedged tranche has the <u>same</u> correlation sensitivity as the tranche itself. Hence, correlation sensitive tranches can be combined with underlying CDS and index positions without altering the correlation behaviour of the strategy.

Chart 14: Rho as a Function of Subordination (Continuous CDO Example)



We divide Rho by the Notional size of each Continuous CDO Tranche (€5mn).
Source: Merrill Lynch.

Back to Table of Contents

6. Spread Convexity

Spread convexity for a credit product represents the curvature of its MTM as a function of the underlying spreads. The spread convexity across tranches is significantly different reflecting the different spread risks associated with each tranche. In addition, the spread convexity can be relatively large for a tranche compared to a single-name CDS or a CDS index. It is this difference in spread convexity across tranches and the underlying single-name CDS that makes delta-hedged portfolios exposed to spread convexity risk.

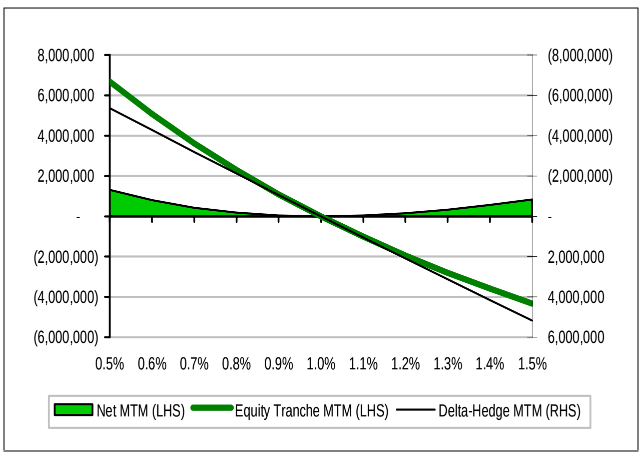
Behaviour of Delta-Hedged Tranches

Equity and senior tranches have different spread sensitivity relative to delta-hedged portfolio

Chart 15 and Chart 16 highlight the MTM behaviour of a delta-hedged long equity tranche and a delta-hedged long senior tranche respectively as the underlying CDS spreads shift uniformly away from their initial spread of 100 bps. The shaded portion of the graphs represents the net MTM. We observe the following:

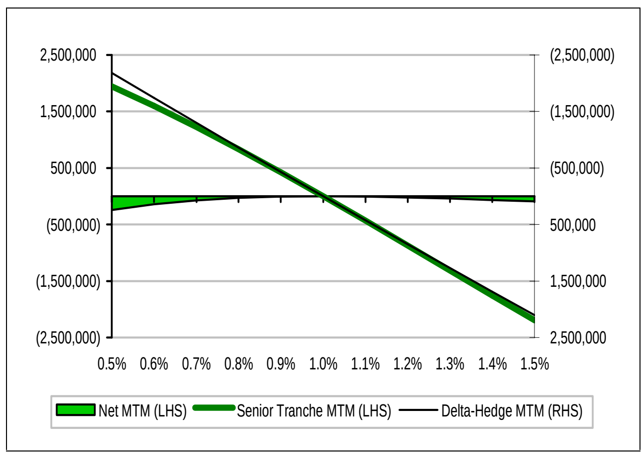
- For a uniform shift in spreads, the equity tranche is more convex than its delta-hedge resulting in a net positive P&L.
- For a uniform shift in spreads, the senior tranche is less convex than its delta-hedge resulting in a net negative P&L. In fact, the senior tranche in the chart below is concave.

Chart 15: Delta-Hedge Long Equity Tranche as a Function of Uniform Shift in All Spreads



Source: Merrill Lynch

Chart 16: Delta-Hedged Long Senior Tranche as a Function of Uniform Shift in All Spreads



Source: Merrill Lynch

We infer from the above charts that while **delta-hedged tranches are MTM neutral for small spread changes they are sensitive to larger CDS spread moves**. This is because Delta is itself a function of spreads and changes as spreads move. A constant rebalancing of deltas is required for the delta-hedged position to remain spread-neutral. Rebalancing of deltas leads to gains and losses based on the investor's exposure.

In order to understand the intuition behind spread convexity and the resulting direction of the MTM of a delta-hedged position, let us consider the behaviour of two portfolios: (a) a delta-hedged long equity tranche (long correlation) and (b) a delta-hedged long senior tranche (short correlation). We assume that all spreads move together (either all up or all down) (see Table 5).

Table 5: Delta-Hedged Portfolio P&L for Uniform Change in All Spreads

	All Spreads Widen	All Spreads Tighten
Long Correlation		
Sell Protection on Equity Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of All Credits	-	+
Average Delta Over Spread Movement	-	+
Effective Delta Hedge	Overhedged	Underhedged
Net P&L (or net MTM)	+ MTM	+ MTM
Short Correlation		
Sell Protection on Senior Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta	+	-
Average Delta Over Spread Movement	+	-
Effective Delta Hedge	Underhedged	Overhedged
Net P&L (or net MTM)	- MTM	- MTM

Assume all spreads in delta-hedged portfolio are initially trading at similar levels.
Source: Merrill Lynch

For a delta-hedged long equity portfolio, an **increase in all the CDS spreads together** implies a lower Delta (see earlier section). Therefore, the average Delta for the tranche decreases over the spread increase implying that the tranche is overhedged. Therefore, the hedge MTM is greater in magnitude than the MTM of the tranche. Since the MTM on the hedge is positive, we infer that the net MTM is positive. From the perspective of maintaining a hedged portfolio and rebalancing Deltas, the investor would need to sell additional CDS at higher spreads thus locking in a profit. We use similar intuition to understand the P&L of any other delta-hedged portfolio for a uniform shift in spreads.

In Table 5 we have assumed that all CDS spreads shift uniformly in the same direction (or *positive* spread correlation). Consider the situation when spreads are uncorrelated, e.g. **one CDS spread moves and the others remain constant**. Table 6 highlights the net P&L of a delta-hedged portfolio as one credit either widens or tightens while the other spreads remain constant.

Table 6: Delta-Hedged Portfolio P&L for Change in One Credit Spread

	One Spread Widens	One Spread Tightens
Long Correlation		
Sell Protection on Equity Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of Credit *	+	-
Average Delta Over Spread Movement	+	-
Effective Delta Hedge	Underhedged	Overhedged
Net P&L (or net MTM)	- MTM	- MTM
Short Correlation		
Sell Protection on Senior Tranche	- MTM	+ MTM
Buy CDS (Delta notional)	+ MTM	- MTM
Change in Delta of Credit *	-	+
Average Delta Over Spread Movement for the Credit	-	+
Effective Delta Hedge	Overhedged	Underhedged
Net P&L (or net MTM)	+ MTM	+ MTM

* The other credit deltas also change. However, since those credit spreads remain constant, they do not impact the MTM of the CDS leg and as a result do not contribute to the net P&L of the delta-hedged portfolio.
Assume all spreads in delta-hedged portfolio are initially trading at similar levels.
Source: Merrill Lynch

Refer to important disclosures on page 38.

In the case of a delta-hedged long equity tranche scenario, we note that the Delta increases (decreases) for the one credit that widens (tightens)[10]. The other deltas also change with a move in one spread but the MTM of the Delta hedge is only due to the CDS whose spread has moved. Thus the other deltas are not required in the MTM analysis in Table 6. The intuition used to generate the net P&L is similar to that described above.

Realised Correlation

As spreads of the underlying credits move the resulting gain or loss of the delta-hedged tranche is a function of the "realised" correlation. **Realised correlation is defined as the observed (spread) correlation between the credits in a portfolio relative to the assumed underlying (default) correlation of the portfolio**. This assumed underlying default correlation is used to price the tranche and determine the deltas for each credit.

Realised correlation can be either positive or negative . . .

The same movement in spreads can represent either positive or negative realised correlation depending on the correlation of the underlying portfolio. For example, if all spreads move together with an observed correlation of 60%, then the realised correlation would be positive relative to a tranche that is priced at a correlation of less than 60%. The realised correlation, however, would be negative relative to a tranche priced at a correlation of more than 60%.

. . . and tends to have a positive bias in a tight spread environment

If we ignore company-specific events, credits spreads tend to be positively correlated for small spread moves resulting in a positive realisation of correlation[11]. This positive spread correlation bias tends to get reinforced in the current environment of tight spreads. However, a default (or a distressed widening) by one of the credits combined with a limited movement in spreads of the other credits in the portfolio would be reflected as a negative realisation of correlation.

Using this terminology, Table 5 and Table 6 highlight positive and negative realisations of correlation respectively. We observe that the same delta-hedged tranche does not always generate a positive P&L when spreads change, i.e. it is **not unambiguously long convexity**. However, a delta-hedged tranche is **unambiguously long or short correlation** depending on the subordination of the tranche and whether the investor is long or short the tranche. Net P&L of a delta-hedged tranche is clearly a function of realised correlation as shown in the tables above.

We observe the following convexity biases:

A delta-hedged tranche that is **long correlation** will generate a gain for a positive realised correlation and a loss for a negative realised correlation.

A delta-hedged tranche that is **short correlation** will generate a loss for a positive realised correlation and a gain for a negative realised correlation.

Gamma/iGamma/nGamma

Gamma is a second-order risk measure

In order to quantify the convexity biases described above, we define the following three convexity measures **for delta-hedged portfolios** that take into account positively correlated, uncorrelated and negatively correlated spread moves respectively:

- **Gamma**: is defined as the portfolio convexity corresponding to a uniform relative shift in all the underlying CDS spreads.
- **iGamma** (individual Gamma): is defined as the portfolio convexity resulting from one CDS spread moving independently of the others, i.e. one spread moves and the others remain constant.

[10] The reasoning is explained in an earlier section.
[11] Assume a default correlation of around 20% in this example.

- **nGamma** (negative Gamma): is defined as the portfolio convexity corresponding to a uniform relative shift in underlying CDS spreads, with half of the credits widening and half of the credits tightening by a uniform amount.

The spread movements relating to **Gamma reflect a positive realisation of correlation** whereas the spread movements corresponding to the **iGamma and nGamma reflect a negative realisation of correlation**. In other words:

Delta-hedged investors who are **long correlation (**and therefore generate a **gain from positive realised correlation)** will also be **long Gamma** and **short iGamma and nGamma**.

Delta-hedged investors who are **short correlation (**and therefore generate a **gain from negative realised correlation)** will also be **short Gamma** and **long iGamma** and **nGamma**.

Table 7 highlights the Gamma exposures for delta-hedged investors who are either long or short correlation. We see that Gamma of a long senior position can be reduced (or hedged) with a long equity position. Such as Gamma reduction strategy is particularly attractive when Delta hedging becomes expensive, e.g. in volatile market conditions (assuming the spreads move together).

Table 7: Correlation & Convexity

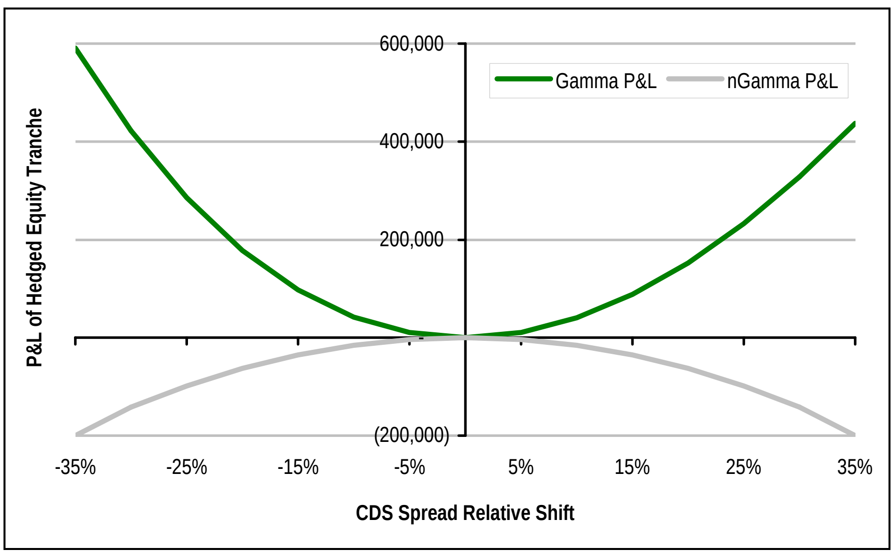
	Delta-Hedged	Convexity		
Correlation View	**Tranche Exposure**	**Gamma**	**iGamma**	**nGamma**
Long Correlation	Long Equity/Short Senior	Long	Short	Short
Short Correlation	Short Equity/Long Senior	Short	Long	Long

Source: Merrill Lynch

Convexity of Equity Tranche

Delta-hedged investors who are long an equity tranche are also long correlation. Therefore as spreads moves they will gain (lose) from any positive (negative) realisation of correlation. In other words they are long Gamma and short iGamma and nGamma. Chart 17 illustrates two of the three measures for this delta-hedged portfolio.

Chart 17: Gamma Profile for a Delta-Hedged Long Equity Tranche



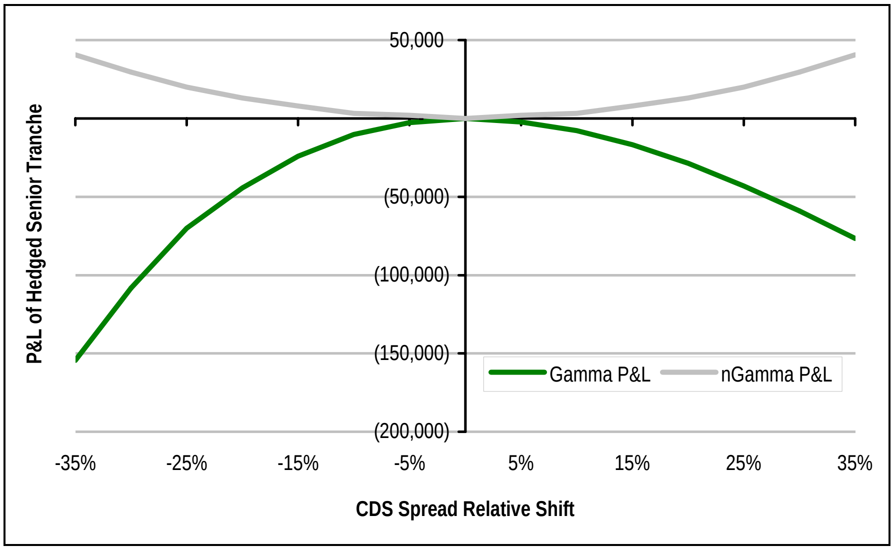
Assume Discrete CDO tranche.
Source: Merrill Lynch

Refer to important disclosures on page 38.

Convexity of Senior Tranche

Delta-hedged investors who are long a senior tranche are also short correlation. Therefore, as spreads moves they will benefit (lose) from any negative (positive) realisation of correlation. In other words they are short Gamma and long iGamma and nGamma. Chart 18 illustrates two of the three measures for this delta-hedged portfolio. Note that Gamma (nGamma) is more convex (concave) for a delta-hedged long equity tranche relative to a delta hedged short senior tranche.

Chart 18: Gamma Profile of a Delta-Hedged Long Senior Tranche

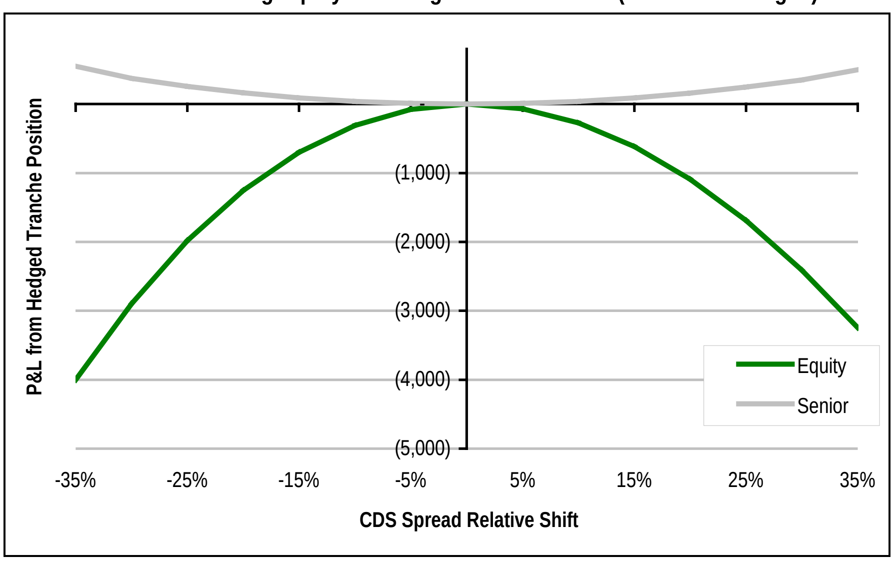


Assume Discrete CDO tranche.
Source: Merrill Lynch

iGamma

Chart 19 highlights the iGamma P&L for the long delta-hedged positions in both the equity and senior tranches. Note that the magnitude of iGamma is smaller than Gamma or nGamma. Also iGamma for a delta-hedged short equity tranche is more convex than the iGamma for the delta-hedged long senior tranche.

Chart 19: iGamma for Long Equity and Long Senior Positions (Both Delta-Hedged)



Assume Discrete CDO.
Source: Merrill Lynch

Back to Table of Contents

7. Instantaneous Default Risk

Another risk factor in correlation products is the risk of an instantaneous default to a recovery value. For unhedged tranches this risk is high, for delta-hedged tranches this risk is somewhat lower but can still be sizeable depending on the tranche in question. The P&L following an instantaneous default depends on whether the tranche is long or short correlation.

Default-to-Recovery

Hedged or unhedged tranche positions are exposed to instantaneous default risk . . .

Unhedged tranche positions are sensitive to (rare) instantaneous defaults and the idiosyncratic risk of the underlying portfolio. For unhedged tranche positions, this is a first-order risk, for hedged tranche positions this is a second-order risk.

We define the Default-to-Recovery or DTR as the net P&L of a tranche position (hedged, or unhedged) resulting from an instantaneous default of a credit, keeping all other credit spreads in the portfolio unchanged.

. . . which is typically a negative realisation of correlation

An instantaneous default is typically a **negative realisation of correlation**, unless spreads on the rest of the undefaulted names in the portfolio widen substantially once default occurs. In fact, DTR can be viewed as the most severe form of iGamma so a tranche that is short iGamma is also short DTR, and a tranche that is long iGamma is also long DTR. Therefore, DTR should typically benefit delta-hedged tranches that are short correlation. In other words:

- **Short correlation** Delta-Hedged tranches are **long DTR**.
- **Long correlation** Delta-Hedged tranches are **short DTR**.

We examine the impact of DTR with and without spread changes for the Discrete CDO example. In Table 8 we compute the net P&L after a DTR in the underlying portfolio for a long Delta-hedged equity position and a long Delta-hedged senior position. We assume that defaults recover 35% i.e. the loss amount is €6.5m.

The DTR for the Delta-hedged long equity position is negative, although the magnitude of the loss *falls* if spreads simultaneously *widen*, and the magnitude of the loss *rises* if spreads simultaneously *tighten*.

The DTR for a Delta-hedged long senior position is positive, although the magnitude of the gain *falls* if spreads simultaneously *widen*, and the magnitude of the gain *rises* if spreads simultaneously *tighten*.

For a DTR of one credit accompanied by a simultaneous shift in spreads of the remaining credits, we observe the following:

- If spreads on the remaining credits widen, the realised correlation would be higher. Therefore, a *long correlation* position would *lose less* and a *short correlation* position would *gain less*.
- If spreads on the remaining credits tighten, the realised correlation would be lower. Therefore, a *long correlation* position would *lose more* while a *short correlation* position would *gain more*.

For a long equity tranche position, following a DTR with no simultaneous spread change the investor has to pay the loss payments of €6.5m under the portfolio default swap. However, the investor has a positive P&L from settling the single-name CDS hedge, but this is less than the loss payment under the portfolio default swap as the Delta of the credit is less than 100%. In addition, the investor also incurs a negative MTM on the tranche position as the tranche width is reduced.

For a long senior tranche position, following a DTR with no simultaneous spread change, the investor is not exposed to any loss payments on the portfolio default swap, as this is only an exposure for an investor in the equity (first-loss) tranche. The investor again has a positive P&L from settling the single-name CDS hedge (although the magnitude of this payment is lower than in the equity tranche example as a senior tranche Delta is smaller than an equity tranche Delta). Finally, the investor again incurs a negative MTM on the tranche position as subordination is reduced.

Table 8: P&L From Instantaneous Default

One Credit	All Other Credits	P&L Of Delta-Hedged Long Position: Equity (0%-4%) Tranche (mn)	Senior (8%-12%) Tranche (mn)
Defaults	50 bps Widening	(2.86)	0.24
Defaults	25 bps Widening	(4.13)	0.42
Defaults	15 bps Widening	(4.57)	0.49
DTR	No Spread Change	(5.14)	0.60
Defaults	15 bps Tightening	(5.57)	0.69
Defaults	25 bps Tightening	(5.76)	0.73
Defaults	50 bps Tightening	(5.79)	0.74

Source: Merrill Lynch

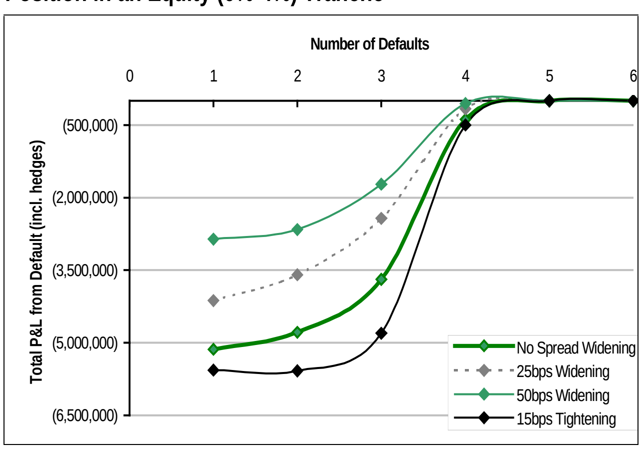
As a result of the different mechanics, the **Equity DTR is about 9x larger than the Senior DTR**. Hence, **selling (buying) protection on a senior tranche can be an effective way to hedge DTR exposure to a long (short) equity position**

We also note that mezzanine tranches are more neutral with respect to DTR. In the same way that we can construct a correlation neutral mezzanine tranche, it is possible to construct attachment points such that a mezzanine tranche has zero exposure to DTR.

DTR P&L For Multiple Defaults

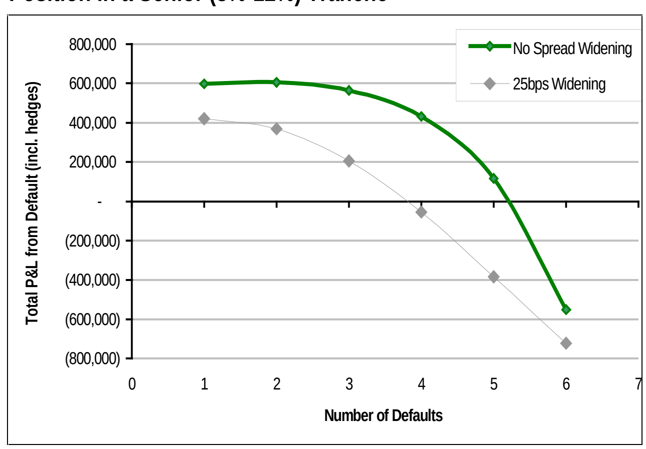
We can further investigate the DTR dependency of a delta-hedged tranche by looking at the P&L profile following multiple defaults with and without simultaneous spread changes. In Chart 20 and Chart 21 we plot the marginal net P&L of the delta-hedged equity and delta-hedged senior tranches following multiple defaults in the underlying portfolio.

Chart 20: Marginal DTR P&L, Including Hedges, of a Long Position in an Equity (0%-4%) Tranche



Source: Merrill Lynch

Chart 21: Marginal DTR P&L, Including Hedges, of a Long Position in a Senior (8%-12%) Tranche



Source: Merrill Lynch

For a delta-hedged long equity tranche position, the marginal P&L impact is usually always negative, unless spreads simultaneously widen significantly, but decreases as multiple defaults occur and the equity tranche is increasingly eroded. This scenario represents a low correlation event. If remaining spreads simultaneously widen, it represents a slightly higher correlation event and the P&L on each default is less negative since the position is long correlation.

For a delta-hedged long senior tranche position, the marginal P&L impact is usually always positive, unless spreads widen significantly, but decreases as multiple defaults occur and the senior tranche begins to behave more like a mezzanine tranche. This scenario represents a low correlation event. If remaining spreads simultaneously widen, it represents a slightly higher correlation event and the P&L on each default is less positive since the position is short correlation.

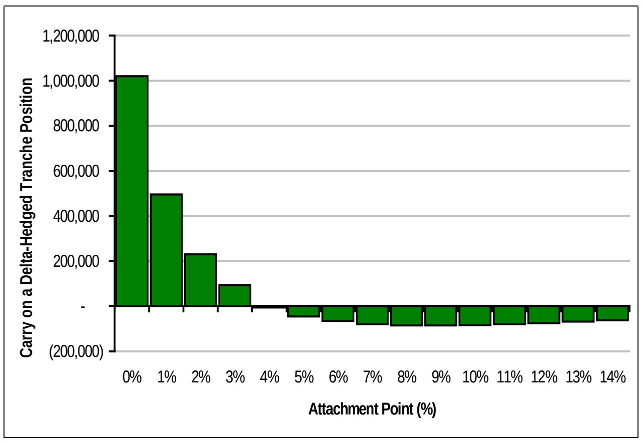
Implications of DTR on the Carry of a Delta-Hedged Tranche

A delta-hedged position is not a carry neutral trade for equity or senior tranches

The carry on the **delta-hedged equity tranche** is typically positive for an investor who is long the tranche and delta-hedged by buying protection on the underlying credits. (Conversely, this represents a negative carry position for a dealer who has bought protection on the tranche and delta-hedged by selling protection on the underlying names.) However, the carry position for the investor becomes negative for tranches that are more senior in the capital structure (Chart 22).

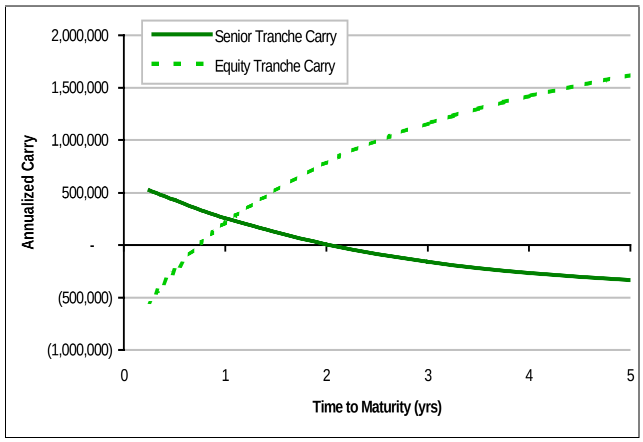
A delta-hedged tranche is not a risk-free position i.e. delta-hedging a tranche is not a perfect static hedge over the life of the transaction. An investor still has exposure to DTR. As was shown above, DTR has a *negative* P&L impact for a long position in the Equity tranche and a *positive* P&L impact for a long position in the senior tranche[12]. **Thus a delta-hedged tranche position should not be expected to be a carry neutral trade**.

Chart 22: Carry P&L on a Long Position in a Delta-Hedged CDO as a Function of Subordination (Continuous CDO Example)



Source: Merrill Lynch

Chart 23: Senior and Equity Tranche Carry as a Function of Time to Maturity



Source: Merrill Lynch. Long position in Delta-hedged tranches.

The carry changes over time

Moreover, the net carry will change over time due to time decay and the rebalancing of Deltas. For instance, a long position in a 5yr delta-hedged senior tranche initially has a negative carry. However, as time to maturity approaches (and assuming no defaults in the underlying portfolio), the Deltas tend to 0% and so the carry on the trade increases. Similarly, a long position in a 5yr delta-hedged equity tranche initially has positive carry but as time to maturity approaches, the Deltas tend to 100% and the carry on the trade turns negative (Chart 23).

A delta-hedged tranche position that is *long correlation* is *short DTR* and hence is a *positive carry* trade at inception.

A delta-hedged tranche position that is *short correlation* is l*ong DTR* and hence is a *negative carry* trade at inception.

[12] To hedge against this default risk, an investor could instead buy protection on the individual credits in the full notional amount of that credit, However, this would then be an overhedge for the spread risk of the tranche.

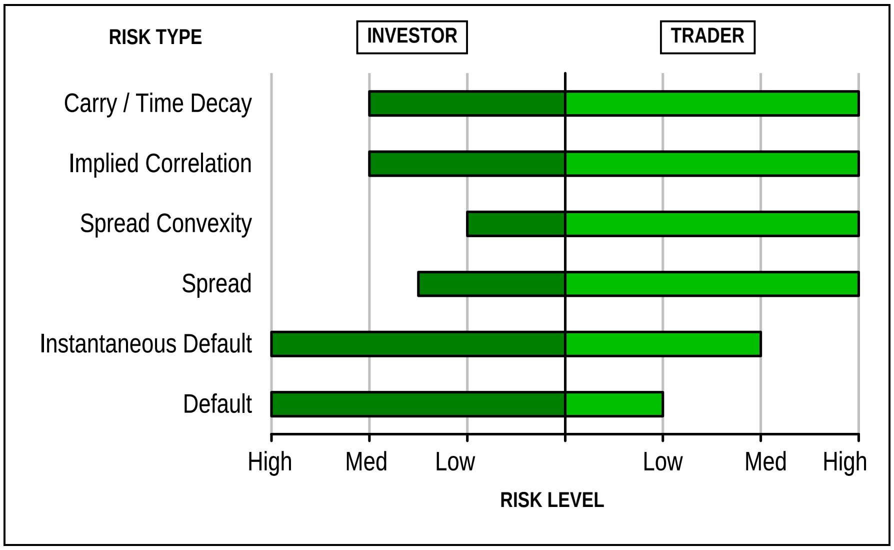
Back to Table of Contents

8. Correlation Strategies

Risk Exposure for Correlation Investors and Traders

Participants in the correlation market are exposed to several types of risks. The key risks include **default risk, instantaneous default risk, spread risk, spread convexity risk, implied correlation risk** and **time decay**. In general, we believe that the risk exposures for correlation investors are slightly different from those faced by correlation traders. We highlight this in Chart 24.

Chart 24: Key Risks Faced by Correlation Investors and Traders



Source: Merrill Lynch

Correlation investors more exposed to default and instantaneous default risk than traders . . .

Correlation investors typically have outright long positions and have a relatively high exposure to both default risk and instantaneous default risk. Correlation traders on the other hand who hold delta-hedged tranches have lower exposure to default risk. However, they remain somewhat exposed to the risk of instantaneous default (or DTR) depending on whether they are long or short correlation and the tranche to which they are exposed. As shown earlier, a delta-hedged equity tranche has greater DTR exposure relative to a delta-hedged senior tranche.

. . . though traders bear more spread, spread convexity . . .

The spread risk and the spread convexity risk borne by correlation traders is higher than that borne by correlation investors. Spread convexity in fact dictates some of the strategies for correlation traders. Correlation investors, however, do face spread risk which results in MTM volatility. However, if the fundamental credit view plays out correctly over time then this MTM would move in the desired direction.

. . . correlation . . .

Correlation investors are also exposed to the MTM volatility arising out of changes in implied default correlation of the underlying tranche. However, correlation traders with delta-hedged portfolios maintain a relatively high exposure to implied correlation. Delta is a function of correlation and consequent rebalancing of deltas can lead to gains and losses for correlation traders based on whether they are long or short correlation.

. . . and time decay risk

Correlation traders also have a high exposure to time decay. This is again a result of deltas changing with the passage of time. As discussed earlier, equity tranche deltas increase and senior tranche deltas decrease as we approach maturity (assuming no defaults). Correlation investors are also exposed to time decay though to a lower extent. The MTM arising from time decay can be attributed to a change in expected loss for each tranche as we approach maturity.

Expressing a Correlation View

Tranches can be combined with single-name CDS and CDS indices . . .

Correlation traders can isolate the risks described above via a correct combination of different tranches, CDS index (such as iBoxx) and single-name CDS exposures. Short of entering into an exactly offsetting transaction, it is impossible to hedge out all the risks simultaneously. For example, a delta-neutral tranche is spread-neutral (for small changes in spread) but remains exposed to other risks such as instantaneous default, spread convexity, implied correlation and time decay.

. . . to isolate different risks . . .

Correlation traders should be aware of the following **building blocks**:

- A CDS index or a portfolio of single-name CDS has no sensitivity to correlation. Therefore the default correlation of a delta-hedged tranche, for example, is the same as the default correlation of the tranche.
- A single-name CDS or CDS index has some spread convexity[13]. It is, however, dwarfed by the much higher spread convexity of a tranche. Therefore the spread convexity of a tranche is virtually unchanged with the addition of a portfolio of single-name CDS or a CDS index.
- Addition of single-name CDS and CDS index positions to correlation sensitive products, however, does affect the spread sensitivity of the strategy. A delta-hedged tranche, for example, is spread neutral (for small changes in spread) even though each of the components is sensitive to spreads.
- A tranche can be decomposed into Delta equivalent exposures to its underlying credits. The tranche Delta along with Leverage (or Lambda) provide a good measure of credit risk of the tranche.
- Options on index and tranches can be added to create non-linear payoffs.

. . . or express a particular view on correlation

Correlation views can be established in the following manner:

- Investors who believe spreads will be positively correlated should be long Gamma.
- Investors who believe otherwise, i.e. spreads are uncorrelated or negatively correlated, should be long iGamma and nGamma.
- Investors who believe one or more credits can widen substantially while the others remain unchanged should be long DTR.
- Investors who have a view on implied correlation should be either long correlation if they expect implied correlation to increase or short correlation if they expect implied correlation to decrease.

These views can be summarised in Table 9. Note that a tranche that is long Gamma is also short iGamma, nGamma and DTR and vice versa. **Given the current tight spreads, an investor could expect any spread widening to occur together and increase implied correlation. Long correlation/long Gamma trades would benefit from this outcome.**

Table 9: Risk Exposure vs. Correlation View

	Expected Realisation of Correlation	
Risk Exposure for Positive P&L	**Positive**	**Negative**
Gamma	Long	Short
iGamma/nGamma	Short	Long
DTR	Short	Long
Implied Correlation	Long	Short

Source: Merrill Lynch

[13] The spread DV01 (or risky bpv) of a single-name CDS increases as spreads decrease and vice versa. As a result as spreads tighten the net MTM of a short CDS position increases more than the decrease in MTM as spreads widen.

Refer to important disclosures on page 38.

Correlation Trading Strategies

We outline below correlation strategies that express customised views on leverage, correlation, spread, spread convexity, instantaneous default risk and carry.

Strategy 1: Buy Protection on Senior Tranche of iBoxx and Sell Protection on iBoxx Index

Funding a market short and earning positive carry

This trade funds a short on the market and earns a positive carry. This exposure cannot be created using traditional credit instruments. It performs well in a low volatility, tight spread environment due to the positive carry. However, should spreads widen, the trade benefits from being short spread and long spread convexity. This trade creates a position that:

- is short spread DV01,
- is long correlation,
- has high gamma,
- has positive carry.

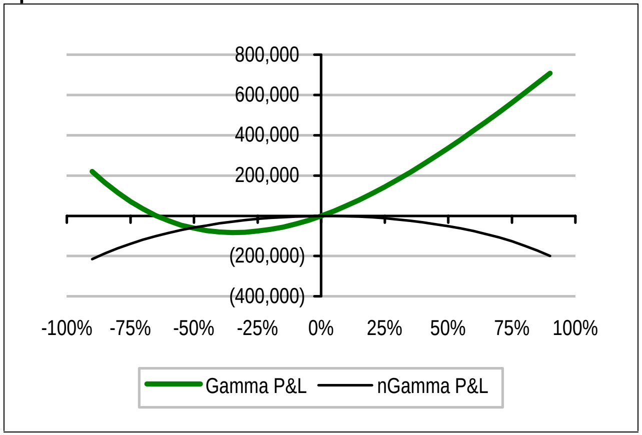
The trade specifics are as follows (all terms are indicative):

- **Buy 5y** protection on **$15mn** of the **7-10% iBoxx** tranche at **120bps** (paying $180k per year).
- **Sell 5y** protection on **$50mn** of the underlying **iBoxx index** at **55bps** (earning $275k per year).

The trade has the following key features:

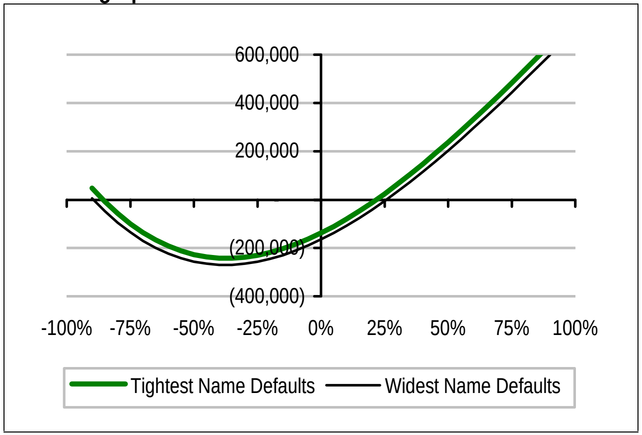
- **Carry**: Net positive carry of $95k per year.
- **Spread DV01**: Short about $11.25k per bp move in iBoxx.
- **Correlation Sensitivity**: Since iBoxx index is insensitive to correlation, the trade's correlation sensitivity is equal to that of the 7-10% tranche.
- **Gamma**: Since the portfolio is not delta-neutral, Gamma refers to the spread convexity of the combined portfolio, i.e. the P&L for a uniform shift in all spreads. This strategy results in a long Gamma profile (Chart 25).
- **Instantaneous Default Risk**: The trade DTR is negative (Chart 26) implying that the index leg loses more than the gain in the senior tranche post an instantaneous default. The trade is long correlation and benefits when all other spreads also widen substantially (i.e. positive realised correlation).

Chart 25: Spread Convexity - P&L vs. Uniform % Shift in All Spreads



Source: Merrill Lynch

Chart 26: Instantaneous Default P&L vs. Uniform % Shift in Remaining Spreads



Source: Merrill Lynch

Strategy 2: Sell Protection on Equity Tranche of iBoxx and Buy Protection on Senior Tranche of iBoxx

Delta-neutral, long Gamma & positive carry trade

This delta-neutral trade is long Gamma and benefits from underlying spreads either all widening or tightening simultaneously. In an environment of stable spreads, the position would benefit from a positive carry. This trade creates a position that:

- is long correlation,
- is highly convex,
- is Delta neutral,
- has positive carry.

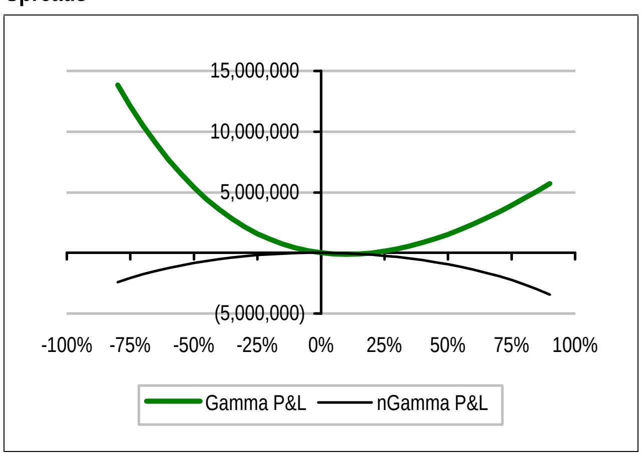
The specifics of the trade are as follows:

- **Sell 5y** protection on **$45mn** of **0-3% iBoxx** tranche at **41% upfront + 500bps** running (earning $18.45mn upfront plus $2.25mn per year).
- **Buy 5y** protection on **$135mn** of the **7-10% iBoxx** tranche at **120 bps** (paying $1.62mn per year).

The trade has the following key features:

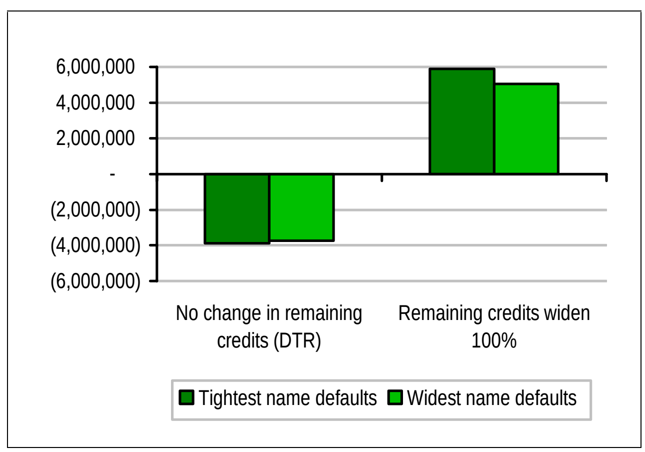
- **Carry**: Net carry of $18.45mn upfront plus $630k per year.
- **Spread DV01**: is zero as the position is structured to have offsetting price changes for small changes in the iBoxx index.
- **Correlation Sensitivity**: Both legs of the trade are individually long correlation so the combined trade is also long correlation.
- **Gamma**: This delta-neutral long correlation trade results in a long Gamma position that is highly convex (Chart 27).
- **Instantaneous default risk**: The DTR of the trade (Chart 28) is negative reflecting the relatively large negative DTR of the 0-3% tranche compared to the positive DTR of the 7-10% tranche. The negative DTR also explains why the delta-neutral trade has a positive carry.

Chart 27: Spread Convexity - P&L vs. Uniform % Shift in Spreads



Source: Merrill Lynch

Chart 28: Instantaneous Default P&L



Source: Merrill Lynch

Refer to important disclosures on page 38.

Strategy 3: Buy Protection on 10-15% iBoxx Tranche

Cheap short of index

This trade allows the investor to take a leveraged short view relatively cheaply compared to an equivalent short exposure via the iBoxx index. This trade creates a position that:

- is leveraged (Lambda = 2.5x),
- is short spread DV01,
- is long correlation,
- has negative carry.

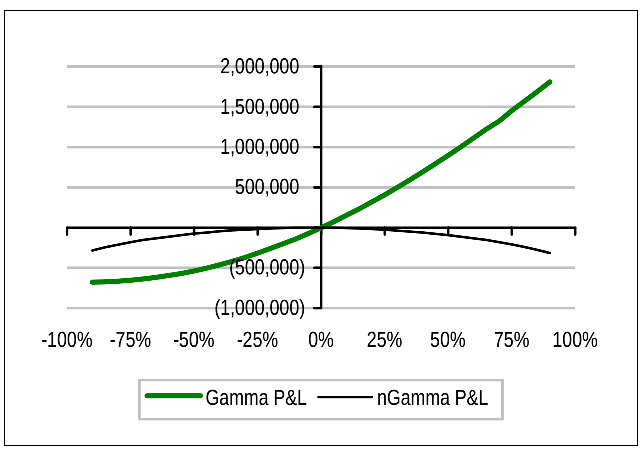
The specifics of the trade are:

- **Buy 5y** protection on **$30mn** of the **10-15% iBoxx** tranche at **60bps** (paying $180k per year).

The trade has the following key features:

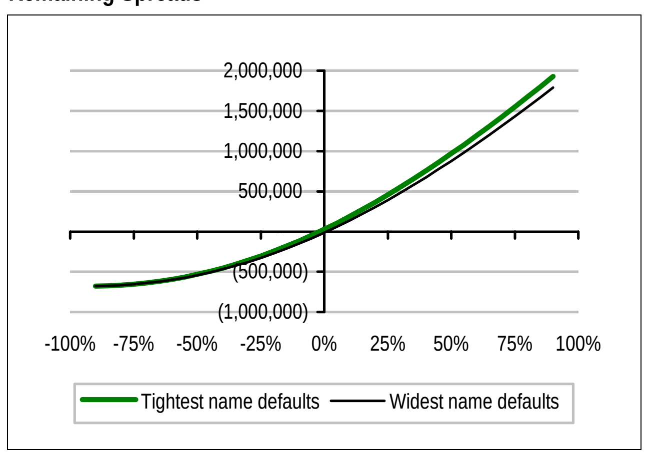
- **Carry**: Negative carry of $180k per year
- **Spread DV01**: short around $40.5k per bp move in the iBoxx. To get the same short DV01 via the index (offered at 55bps), the investor would have to buy around $90mn of protection resulting in a carry of $495k per year. The **cost/leverage ratio** for this short tranche position (60/2.5 = 24) is less than half the ratio via an index short (55bps/1x = 55).
- **Correlation Sensitivity**: The position is essentially short a senior tranche and therefore long correlation. The trade would benefit from an increase in implied correlation.
- **Gamma**: The Gamma P&L in this case refers to the curvature of the tranche MTM for uniform shift in spreads. As seen in Chart 29, the tranche position has a highly convex short spread profile.
- **Instantaneous default risk**: Chart 30 highlights that the P&L following an instantaneous default is similar to the Gamma P&L in Chart 29. This suggests that the default P&L is dwarfed by the P&L resulting from the spread convexity of the remaining credits.

Chart 29: P&L vs. Uniform % Shift in Spreads



Source: Merrill Lynch

Chart 30: Instantaneous Default - P&L vs. Uniform % Shift in Remaining Spreads



Source: Merrill Lynch

Back to Table of Contents

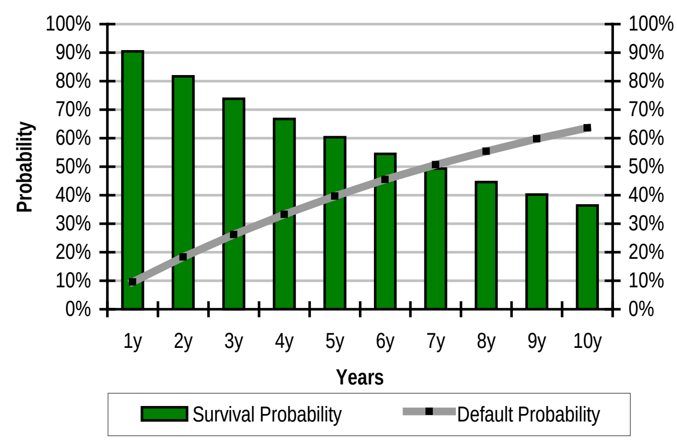
Appendix: Copula Functions

From Single-Name to Multi-Name

Bootstrap procedure to compute single-name survival curve is relatively straightforward . . .

A common approach to compute default or survival probabilities[14] for an individual credit is to use a bootstrap procedure based on CDS quotes and an exogenous (that is, not derived by the model itself) assumption for the recovery rate. We have discussed this method in some detail in an earlier report[15]. Chart 31 highlights this term structure for a generic issuer.

Chart 31: Survival Curve and Default Probabilities



Assume flat CDS curve of 300 bps from years 1-10 and a recovery of 50%.
Source: Merrill Lynch

The survival curve can be used to compute the probability of default (or survival) at each point in time for the issuer. For example, Chart 31 implies that the probability of not triggering a credit event before 3 years (or the probability of surviving at least 3 years) is approximately equal to 74%.

We use the following notation:

- Let τ_j denote the time until default for the j^{th} issuer.
- Let $S_j(t)$ denote the survival probability function at each point in time, t.
- Let $F_j(t)$ denote the default probability function at each point in time t.

The probability that the issuer will survive at least t years is given by:

$$S_j(t) = \Pr(\tau_j > t)$$

The probability that issuer will default within t years is given by:

$$F_j(t) = \Pr(\tau_j \le t)$$

We can easily see that:

$$S_j(t) = 1 - F_j(t).$$

[14] Survival Probability = 1 - Default Probability
[15] Refer to ML report titled *"Default Swap Unwinds"* by Martin/Francis/Kakodkar dated 11 July 2002.

. . . but computing a joint survival curve for multi-name credit derivatives is not

Aggregating individual survival curves into a joint survival curve for all names in the portfolio is the fundamental problem with multiname credit derivatives.

Let us suppose we have a basket default swap referencing a portfolio of three issuers. We can then ask the following questions:

- How do we compute a "portfolio" based or joint survival probability $S_{portfolio}(t)$ such that all credits survive at least the next two years?
- How do we measure the likelihood of joint occurrences of default within a two year horizon?

We use the following added notation:

- Let $S_{portfolio}(t)$ denote the joint survival probability function at time, t.
- Let $F_{portfolio}(t)$ denote the joint default probability function at time t.

The probability that all three credits in the basket will survive at least 2 years is given by:

$$S_{portfolio}(2y) = 1 - F_{portfolio}(2y) = \Pr(\tau_1 > 2y, \tau_2 > 2y, \tau_3 > 2y)$$

Unfortunately **there is no explicit solution to this problem**, that is, it is not possible to retrieve this multivariate cumulative probability from market quotes. This happens for two main reasons:

- Lack of reliable data due to the lack of liquid traded portfolio assets such as NTD baskets.
- The dependence on default correlation which is a difficult parameter to determine (as explained earlier in the report).

Therefore, our problem can be summarised as follows:

Individual CDS based information

$$S_1(t) = \Pr(\tau_1 > t)$$
$$S_2(t) = \Pr(\tau_2 > t) \quad \Rightarrow \quad S_{portfolio}(t) = \Pr(\tau_1 > t, \tau_2 > t, \tau_3 > t)$$
$$S_3(t) = \Pr(\tau_3 > t)$$

Portfolio based information

Copula Functions To The Rescue

Linking Individual Distributions to One Joint Distribution

Copula functions provide an efficient link between multiple single-name survival curves to one multi-name survival curve

The number of joint default events increases exponentially as a function of the number of credits. As a result, for typical multiname portfolios of 50-100 credits, the computation of joint default probabilities becomes increasingly complex and requires efficient numerical methods. This is where copula functions come to the rescue.

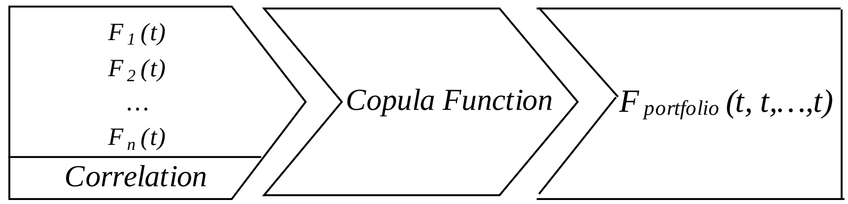
In fact, **copulas provide an efficient way to link multiple unidimensional survival curves with a single multidimensional survival curve**. This fact derives from an important theorem, attributed to Sklar (1959), which, in our context, can be stated as follows:

Theorem (Sklar): Let $F_{portfolio}(t)$ be the "portfolio" default distribution function and $F_j(t)$ the default distribution function for the generic j^{th} issuer with $j=1,2,\ldots,n$, where n stands for the number of obligors referenced by the contract. Then there exists an n-dimensional **copula function** C such that:

$$F_{portfolio}(t) = \Pr(\tau_1 < t, \tau_2 < t, \ldots, \tau_n < t) = C[F_1(t), F_2(t), \ldots, F_n(t)]$$

Sklar's theorem provides the core idea of dependency modeling via copula functions. It states that for any "portfolio" multidimensional distribution function, the single obligors' distribution functions $F_j(t)$ and the dependency structure (represented by the default correlation) can be **separated**, via a copula function. Conversely, the theorem shows that, starting from the univariate survival curves for each obligor, it is possible to **aggregate** them into a multidimensional survival curve by using a copula function.

Chart 32: Linking Individual Curves



Source: Merrill Lynch

Because of its analytical tractability and the small number of parameters required, **Gaussian (or normal) copula represents the current market standard** in modeling portfolio credit risk sensitive positions and their relative trades. The types of portfolios include CDOs as well as NTD-basket default swaps and notes.

Inverted Distribution

Before providing an explicit expression for the Gaussian copula, we need to introduce the concept of the inverted distribution function to understand the simulation algorithm that is presented in the next section.

We know that $F(t)$ maps time t to the probability of default. Now suppose we have the opposite situation, i.e. we are given the probability of default and asked to compute the implied time t. **The function that maps the cumulative default probability to the corresponding time t, is called the inverted distribution function** and is a key concept in the copula framework.

Using F^{-1} to denote the inverted distribution function, we have:

$$t = F^{-1}[F(t)]$$

From Sklar's Theorem, we derive the following relationship:

$$C[F_1(t), \ldots, F_n(t)] = F_{portfolio}\{F_1^{-1}[F_1(t)], \ldots, F_n^{-1}[F_n(t)]\}$$

Gaussian Copula

Gaussian Copula is the market standard

Let N^{-1} be the inverted **one-dimensional** Gaussian or normal distribution function for an issuer. The **multidimensional** Gaussian copula C^G is given by:

$$C^G[F_1(t), \ldots, F_n(t)] = N_R\{N^{-1}[F_1(t)], \ldots, N^{-1}[F_n(t)]\}$$

where N_R is the multidimensional Gaussian distribution function with correlation matrix R. From Sklar's Theorem, we have the following:

$$F_{portfolio}(t) = N_R\{N^{-1}[F_1(t)], \ldots, N^{-1}[F_n(t)]\}$$

Refer to important disclosures on page 38.

Simulation Algorithm

How can we simulate the time-until-default random variables taking into account both the individual creditworthiness (individual measure) and the default correlation (portfolio measure)?

David X. Li, in his paper "*On Default Correlation: A Copula Function Approach*" dated 1999, offers an interesting way to perform this task using the copula framework we discussed above:

1. Simulate the *n* random variables $Y_1, Y_2, \ldots, Y_n$ from a multidimensional normal distribution function[16] with correlation matrix R.
2. Map back the Y's to the τ's using $\tau_j = F_j^{-1}\left[N(Y_j)\right]$

The tricky part of this algorithm is represented by the correct specification of the single-name distribution function $F_j(t)$. We are going to assume that default distribution functions follow an exponential distribution with parameter h[17], i.e.

$$F(x) = 1 - e^{-hx}$$

This basic assumption allows us to provide an explicit solution for step 2 which is given by the following expression[18]:

$$\tau_j = -\frac{1}{h}\ln\left[1 - N(Y_j)\right]$$

This is an important result as it allows us to obtain the sample path of the default arrival time for each issuer by taking into account both the key inputs:

- Individual creditworthiness via the factor h which depends on the underlying CDS curve of the issuer, and
- Correlation matrix R which affects the sampling procedure of the random variables Y's.

---

[16] This task consists in a decomposition of the correlation matrix R (such as the Cholesky decomposition) and the subsequent generation of correlated normal random variables. We refer the reader to Press et al., (1993), "*Numerical Recipes in C: The Art of Scientific Computing*"

[17] The parameter h is one of the main building block of modern credit risk management models. It is commonly called *hazard rate* and is defined as $h(t) = \lim_{\Delta t \to 0} \Pr\left[t < \tau \le t + \Delta t \mid \tau > t\right]$. It represents the probability that the issuer will survive for an additional infinitesimal period Δt conditional on the fact that it has survived until time t. By assuming a flat term structure of credit default swap quotes, it is possible to show that the relation $h = \frac{CDS\ spread}{1 - \mathrm{Re}\cos very}$ holds. In more sophisticated models h is a stochastic variable usually recovered from market data by assuming a certain specification of the "jump" process used to model the occurrence of defaults.

[18] We can easily work out this expression by using the definition of inverted distribution function: if $F(x) = 1 - e^{-hx} = W$, where W lies between 0 and 1. Then, we have $F^{-1}(W) = -\frac{1}{h}\ln(1 - W)$.

A Simple Application

We use the above methodology to compute the portfolio default distribution function for a portfolio with the following characteristics:

- Reference pool of 5 credits;
- Constant pairwise correlation of 50%;
- Flat CDS spread of 200bps; and
- Recovery rate of 50% for each credit.

Table 10 highlights the output of the simulation algorithm by showing default arrival times for each of the issuers; for example, looking at the first simulation run, we can see that the first default occurs in year 58 by issuer five.

The first default occurs in year 58 by issuer 5

Table 10: Copula Simulation of Default Arrival Time (Years)

Simulation #	Issuer 1	Issuer 2	Issuer 3	Issuer 4	Issuer 5	First to Default Time
1	122.3	100.6	112.9	61.4	57.8	57.8
2	149.1	6.6	98.4	128.8	112.6	6.6
3	4.6	16.3	68.0	65.7	23.5	4.6
4	326.9	347.8	63.0	424.3	166.4	63
5	74.4	137.2	15.2	77.4	60.8	15.2
…	…	…	…	…	…	…
…	…	…	…	…	…	…
9996	35.4	69.8	80.7	32.2	29.8	29.8
9997	177.1	230.8	173.6	517.0	423.2	173.6
9998	25.7	74.8	260.9	44.7	81.8	25.7
9999	11.2	424.5	140.2	10.3	87.9	10.3
10000	10.5	4.2	46.8	10.5	1.3	1.3

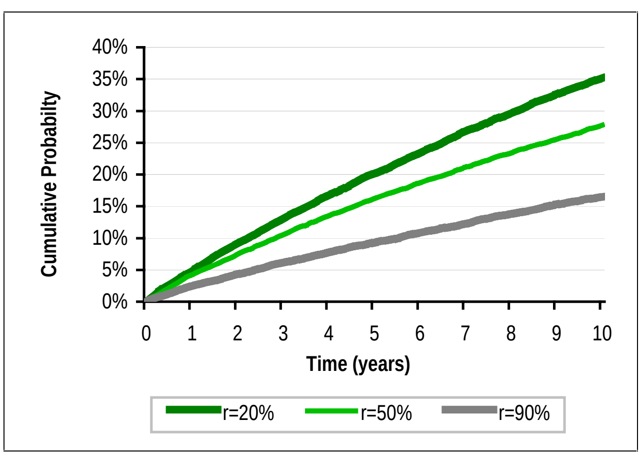
Constant pairwise correlation of 50%, hazard rate *h*=1%, five reference entities, 10,000 simulation runs.
Source: Merrill Lynch.

Using multiple simulations, we plot the portfolio distribution function for different values of correlation (20%,50%, 90%) and seniority (1st and 2nd to default) as shown in Chart 33 and Chart 34.

For example, in the 50% correlation scenario, we see that the probability that at least **one** default occurs within 5 years is around 16%, while the probability of having at least **two** default within 5 years is about 5.25%.

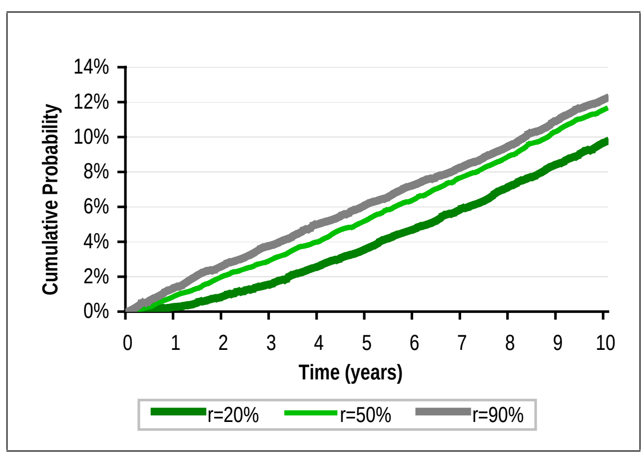
Once the whole portfolio distribution function is computed, it is then possible to calculate the corresponding portfolio losses for each attachment point and fix the breakeven spread for each CDO tranche.

Chart 33: First to Default Distribution Function



Constant pairwise correlation, hazard rate *h*=1%, five reference entities, 50% recovery, 50,000 simulation runs.
Source: Merrill Lynch

Chart 34: Second to Default Distribution Function



Constant pairwise correlation, hazard rate *h*=1%, five reference entities, 50% recovery, 50,000 simulation runs.
Source: Merrill Lynch

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