Common Factors, Information, and Portfolio Choice

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Abstract

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Keywords: Information Economics, REE Models

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Note: All referenced appendices are in the associated Internet Appendix.
Please see: *http://dl.dropbox.com/u/6555606/FactorInfoIA.pdf*

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1 Introduction

How do investors value having asset-specific information compared with having information about common components that affect many assets? How valuable is one's asset-specific information if other investors have common component information relevant for the asset in question? How do market prices differ when one group of investors has all common-component information compared with situations when different groups have information about different common components? While the above three questions are focused on prices, one can ask related questions about investors holdings. In particular, financial economists are interested in knowing which information structures lead investors to have similar portfolios and which structures produce disperse and varied holdings.

To answer the questions in the paragraph above, this paper derives closed-form solutions for asset prices and portfolio holdings in a multi-asset, rational expectations equilibrium (REE) model. The model contains multiple agents (investors) who invest today and consume next period. Throughout the paper, we assume a stock's payoff (dividend) is comprised of three components: an asset-specific component, a part related to common components (factors), and a residual part for which no investor has information. The investors in our model may receive information about asset-specific components and/or information about common components that affect the payoffs of many assets. Modeling the tradeoff faced by investors (asset specific information vs. information about common components) is one aspect that differentiates our paper from existing work.

The inclusion of both asset-specific and common-component information leads to rich and varied portfolio holdings.[1] For example, an investor may have high demand for a stock for which he has asset-specific information. He may also have high demand for the same stock if he has valuable private information about a stock with highly correlated payoffs. When considering common components, an investor may have high demand for a stock (even if

[1] Appendix A provides two additional discussions. First, while this paper's model focuses on different information structures, one can also think of our framework in a more behavioral sense. That is, investors may have certain biases that make some assets appear safer and some appear riskier. Second, we discuss how having both asset-specific information and common-component information is different from simply considering correlated assets. For example, our structure allows investors to consider tradeoffs between asset-specific investment strategies and market-timing strategies.

he does not have asset-specific information) provided he has information about a common component of the stock's payoffs. Of course, having information about a common component is not sufficient to determine if the investor will have high demands for certain stocks. Stocks must load sufficiently on the common component (factor) to outweigh private, asset-specific information that other investors' may have. Finally, and in multi-asset settings, agents balance information about a given asset with a desire to diversify wealth across many assets.

The first contribution of the paper is to solve a model with both asset-specific and common-component information. In doing so, we provide generalized, closed-form solutions to an economic problem that has been outstanding for the past 25 years.[2] To get the closed-form solutions, we make certain assumptions about which group of investors has which information. In this paper, we propose a method to evaluate the restrictiveness of these assumptions—We compare information structures for which we are able to generate closed-form solutions with those for which we must rely on numerical analysis. We conclude that the assumptions are not so restrictive as we generate wider ranges of asset prices and portfolio holdings in closed-form than we generate when looking only at structures that do not admit closed-form solutions.

The second contribution of the paper is to show that our closed-form solutions encompass numerous information structures. For example, we can model the symmetric and complete information structure used in the traditional Capital Asset Pricing Model (also known as the "Frictionless CAPM" or "Full-Information CAPM"). Figure 1 depicts an equity market in which investors are partitioned into four groups. There are four assets and two common components in the economy. Panel A depicts the frictionless/full-info CAPM structure in which each investor group has all possible asset-specific information and common-component information (see the dashed lines).

[Insert Figure 1 About Here]

Figure 1, Panel B depicts one of the 26 different information structure that we model in

[2]See p.653 of Admati (1985). Importantly, we do not solve the mathematical problem posed in Admati (1985) since that problem involves a system of cubic equations. Instead, we use a hybrid model design that combines aspects of information transmission as in Grossman and Stiglitz (1980) with aspects of aggregation of disperse information as in Admati (1985). Our goal is to solve the <u>economic</u> problem posed on p.653 of Admati (1985).

Section 3 (this one is called "Additional Structure $v6$"). In the structure, the investors clearly have different information. Investor groups A and B have asset-specific information about assets 1 and 2 but they do not have any common-component information. Group C has asset-specific information about asset 3 and information about common component f_1. Group D has asset-specific information about asset 4 and information about common component f_2. One cannot readily determine which investor group has the "most" or "best" information.

Generating closed-form solutions gives insights into the effects of information structures on asset prices. For example, we calculate the aggregate market capitalizations (sum of the four assets' prices) for the setting in Figure 1, Panel A and then separately for the setting in Panel B. We then define the difference in market capitalizations as the "information discount factor" or DF_{info}. In other words, DF_{info} is the dollar amount that aggregate prices in the economy are below the full-info CAPM price-levels due to agents having less-than-full information about future payoffs.[3] It turns out that our closed-form solution for DF_{info} can be written as a compact and elegant signal-to-noise expression (though the expression does involve matrices).

The third contribution of our paper is to generate large dispersions in portfolio holdings. Disperse holdings are common in the world's asset markets. For example, 85% of company 401(k) plans in the USA hold between 0% and 10% of their own company's shares.[4] Surprisingly, 5% of 401(k) plans hold between 31% and 50% of their own company's shares. Answering why some companies might hold large positions in their own shares is one goal of this paper. We discuss an answer when concluding in Section 5.

A second example of holdings dispersions concerns the well known home bias phenomenon. Consider a quick study of aggregate mutual fund positions in 467 German stocks as of 31-Dec-2002 (using data from Thomson Financial). From the perspective of the average German stock, foreign funds own 3.66% of the shares outstanding. For one quarter of the German stocks, the same funds hold less than 0.01% of the equity in aggregate. For the

[3]Discount factors are typically expressed as percentages below a reference price. In CARA-normal frameworks, it is common to use price differences rather than price ratios. When parameters are chosen such that the aggregate full-info CAPM market capitalization is one, price differences and price ratios are the same. In our numerical analysis, we also calculate the percentage discount from the full-info CAPM's dollar value. Technically, we could call the discount "$DF_{info\text{-}noise}$". In REE frameworks noise keeps private information from being fully revealed in prices.

[4]Data based on NCEO.org's website.

upper quarter of German stocks, funds hold at least 4.35% of the equity. Foreign ownership dispersion of this magnitude is typical when looking at non-French mutual fund positions in French stocks, non-UK fund positions in UK stocks, and so on. Our paper helps to understand why cross-border ownership of equities may vary considerably across assets. We discuss testable implications and directions for future research when concluding in Section 5.

To the extent that some parts of stocks' payoffs are not perfectly correlated, investors want to diversify across assets. Going back to Figure 1, and assuming assets have similar expected payoffs and factor loadings, the symmetric information structure in Panel A leads each of the four investors to allocate 25% of his portfolio to each of the four assets. There is no dispersion in holdings levels across investors. Under the same payoff and factor loading assumptions, Panel B produces disperse holdings. Investor groups A and B put 33% of their wealth in asset 1 and 33% of their wealth in asset 2. Group C puts 39% of their wealth in asset 3 while group D puts 39% of their wealth in asset 4. Our paper summarizes the degree of holdings dispersion by reporting the root mean squared error (RMSE) calculated using investors' portfolio weights and the full-info CAPM weights. In the case of Panel B, the RMSE is 0.08 indicating that investor-stock weights are typically 8% different from the full-info CAPM weights.

We end this section by noting that our paper nests information structures inspired by existing papers. In addition to some of the papers reviewed below, our framework allows us to analyze new structures (Section 3 models 26 different structures and many others are possible.) Because our model is general, our final contribution is the ability to answer questions such as those posed in the first paragraph of this paper. For example, how do prices and holdings differ when one group of investors has all common component information compared with situations when different groups have information about different common components? The answer to this question is also discussed when concluding in Section 5.

1.1 Literature Review

Our paper is related to theoretical work on information structures, investor holdings, and risk premia. Easley and O'Hara (2004) present a multi-asset model that focuses on the role of public and private signals in determining a firm's cost of capital. Private signals in their model are received only by a group of informed investors as in Grossman and Stiglitz (1980). In our model, it is possible for different groups of investors to have information about different groups of the securities. In this way, investors can be asymmetrically informed without introducing a strict information hierarchy.[5] Bacchetta and van Wincoop (2006) argue in favor of structures with a "[broad] dispersion of information."

In a paper similar in spirit to ours, Hughes, Liu, and Liu (2007) model two groups of investors. Each informed investor effectively observes a global signal "s" and these signals are perfectly correlated across investors. Unlike our paper, investors in the Hughes, Liu, and Liu (2007) paper cannot separate the asset-specific and global components. Additionally, information about the components is not differentially dispersed across investors. Similarly, Kodres and Pritsker (2002) offer a model that contains an underlying factor structure. However, there are no information asymmetries regarding the factors.

Our paper has both important differences from, and certain similarities to, a paper by Albuquerque, Bauer, and Schneider (2009). Their paper contains multiple stocks and multiple time periods. The payoff of a given stock is equal to the sum of three terms: a constant, a local component, and a single global factor. There are public and private signals about both the local components and the global factor. Only one group of investors (from the USA) receives private information about the global factor. Our model contains multiple, global factors, each of which may be known by a different group of investors. Moreover, their model uses a single factor loading (equal to one) for all assets. Our model is more general because it allows for different common factor loadings—both across assets and across factors. Most importantly, American investors in their model have the same informational advantage vis-a-vis each foreign stock. This assumption implies that their model does not generate cross-border holding dispersion based on common-factor information. Our model produces large differences in cross-border holdings (home bias); this dispersion can be tied directly to informational differences about common components.

[5] Our model incorporates an aspect of models which endow all agents with small pieces of information about risky assets payoffs—see Grossman (1976), Hellwig (1980), and Admati (1985). Coval (1997) uses diffuse information in a manner similar to our paper. Van Nieuwerburgh and Veldkamp (2009) study information acquisition and dynamic learning.

Recently, Dumas, Lewis, and Osambela (2010) propose a model to study international portfolio choice when both domestic and foreign investors observe the same public signals but interpret them in a different ways. In their model, the domestic investor is assumed to be more able to understand his own country information. In our model, an investor who has information on a given global factor may have an information advantage about a foreign country's asset (relative to local investors) if this asset loads sufficiently to the global factor.

Finally, two published papers explore learning about categories (common components). Peng and Xiong (2006) allow a representative investor to learn about a stock's market component, industry component, or firm-specific component. While our paper is similar to theirs in this regard, the uses of heterogeneous investors and a rational expectations equilibrium are what differentiates our work. Van Nieuwerburgh and Veldkamp (2010) jointly solve for the information and portfolio allocation problem. The authors mention on p.783 that correlated assets can be factored such that investors learn and invest in risk factors. Our model focuses on the tradeoff when investors simultaneously have asset-specific information and common component information.

The paper proceeds as follows. Section 2 introduces our model, notation, and assumptions. We present general solutions for asset prices and investor portfolios (holdings). Section 3 provides a numerical analysis of prices for 26 different information structures. Section 4 solves for prices and holdings in closed form. We analyze the restrictiveness of the information-structure assumptions needed to produce closed-form solutions. The closed-form solutions give readers economic insights to better understand our model's solutions. We also can solve for the information discount factor in closed-form. The final section concludes.

2 General Model and General Solutions

The model has I investors indexed $i = 1, \ldots, I$ who trade at date 0 and consume at date 1. Each agent i can invest his initial wealth, w_i^0, in a riskless asset and J risky assets indexed $j = 1, \ldots, J$. The riskless interest rate is denoted r_f and we define $R \equiv (1 + r_f)$. For simplicity, we normalize the price of the riskless asset to one. Each risky asset j pays a liquidating dividend $\tilde{P}_j^1$ at date 1. The vector of final payoffs $\tilde{P}^1 = (\tilde{P}_1^1, \ldots, \tilde{P}_J^1)'$ is generated by a K-factor linear process:

$$\tilde{P}^1 = \tilde{\theta} + \mathbf{B}\tilde{f} + \tilde{\varepsilon} \tag{1}$$

The vector $\tilde{\theta} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_J)'$ is the asset-specific component of payoffs, the vector $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_K)'$ contains the K common components (factors), and $\mathbf{B}$ is a $J \times K$ matrix of factor loadings. The remaining part of each asset's final payoff, $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_J)'$, is referred to as residual uncertainty. We assume that $\tilde{\theta}$, $\mathbf{B}\tilde{f}$, and $\tilde{\varepsilon}$ are jointly multivariate normal and independent. We further assume that $\tilde{f}$ and $\tilde{\varepsilon}$ have mean zero. Since $\tilde{\theta}$ is the asset-specific component, we assume its covariance matrix (denoted $\mathbf{\Sigma}_\theta$) is diagonal.[6] For tractability, we assume that the covariance matrix of $\tilde{f}$ is the identity matrix. The covariance matrix of $\mathbf{B}\tilde{f}$ is $\mathbf{BB}'$. Finally, the covariance matrix of $\tilde{\varepsilon}$ is denoted $\mathbf{\Sigma}_\varepsilon$. Table 1 summarizes and describes all variables.

[Insert Table 1 About Here]

The per-capita supply of risky assets is defined as the realization of a random vector $\tilde{z}$. The vector $\tilde{z}$ is independent and jointly normally distributed along with the other variables in the model and has a covariance matrix denoted $\mathbf{\Sigma}_z$. The assumption of random net supply is standard in rational expectations models. As Easley and O'Hara (2004) write "one theoretical interpretation is that this approximates noise trading in the market. A more practical example of this concept is portfolio managers current switch toward using float-based indices from shares-outstanding indices." In order to insure the existence and uniqueness of the date 0 equilibrium price vector, $\tilde{P}^0$, we assume that $\mathbf{\Sigma}_\varepsilon$, $\mathbf{\Sigma}_\theta$, and $\mathbf{\Sigma}_z$ are regular matrices.

We assume all agents have an exponential utility function: $U(\tilde{w}_i^1) = -e^{-a\tilde{w}_i^1}$, where $\tilde{w}_i^1$ is the wealth of investor i on date 1. The utility function has a constant absolute risk aversion with coefficient $a > 0$ which is the same for all agents. The choice of utility functions is also

[6] This assumption is not necessary to solve the model. However, it enables us to distinguish between information about a single asset and information about a common component that affects two or more assets. Note that Table 1 provides descriptions and definitions of all variables used in this paper.

common in rational expectations equilibrium models and ensures that an investor's demand for the risky asset is independent of his initial wealth. Let X_i be investor i's column vector risky-asset holdings. Investor i's final wealth is:

$$\tilde{w}_i^1 = w_i^0 R + X_i'(\tilde{P}^1 - R\tilde{P}^0) \tag{2}$$

2.1 Investors' Information and Model Notation

We partition the I investors in our model into N non-overlapping groups labeled $n = 1, \ldots, N$. Each group of investors represents a fraction, λ_n, of the total number of investors (I) in the market such that $\sum_{n=1}^{N} \lambda_n = 1$.

In our model, investors belonging to the same group n possess the same private information (for asset-specific components and for common components), they face the same optimization problem, and they optimally choose identical portfolios. In this sense they can be said to be identical. We use the following terms interchangeably (and a bit loosely): "investor i from group n", "investor group n", and "investor n". Similar to a Grossman and Stiglitz (1980) framework, we say investor n has asset-specific information about asset j if the investor knows the realization of θ_j. We say investor n has information about common-component k if the investor knows the realization of f_k. To simplify notation, we write the payoffs of the risky assets as:

$$\tilde{P}^1 = \mathbf{C}\tilde{\eta} + \tilde{\varepsilon} \tag{3}$$

Where, $\tilde{\eta} = \begin{pmatrix} \tilde{\theta}' & \tilde{f}' \end{pmatrix}'$ is a $J + K$ column vector and $\mathbf{C}$ is a $J \times (J + K)$ block-diagonal matrix consisting of a $J \times J$ identity matrix, $\mathbf{I}_J$, and the matrix $\mathbf{B}$. The variance-covariance matrix of $\tilde{\eta}$ is $\mathbf{Q} = \begin{pmatrix} \mathbf{\Sigma}_\theta & 0 \\ 0 & \mathbf{I}_K \end{pmatrix}$ where $\mathbf{I}_K$ is the identity matrix of order K.

Definition 1. *For each investor n, we define the diagonal matrix $\mathbf{D}_n$ of order $J+K$. Diagonal elements $1, \ldots, J$ in $\mathbf{D}_n$ correspond to asset-specific components. Diagonal elements $J+1, \ldots, J+K$ in $\mathbf{D}_n$ correspond to common components. We set $\mathbf{D}_n(\cdot, \cdot) = 1$ if investor knows the realization of the associated random variable in $\tilde{\eta}$ and $\mathbf{D}_n(\cdot, \cdot) = 0$ otherwise.*

Definition 2. *We define* $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$*. The matrix* $\mathbf{D}$ *plays an important role in our model as each element on the main diagonal represents the proportion of investors who know the realization of the corresponding random variable in the vector* $\tilde{\eta}$*.*

Definition 3. *For each investor group* n*, the matrix* $\mathbf{M}_n$ *is obtained by eliminating all the null rows of* $\mathbf{D}_n$*. Consequently, the number of rows of* $\mathbf{M}_n$ *is equal to* $J_n + K_n$*, which represents the number of asset-specific and common components about which investor* n *is informed. If investor* n *does not receive any private information,* $\mathbf{D}_n$ *becomes the null matrix and* $\mathbf{M}_n$ *cannot be defined. It is straightforward that* $\mathbf{M}_n'\mathbf{M}_n = \mathbf{D}_n$ *and* $\mathbf{M}_n\mathbf{M}_n' = \mathbf{I}_{J_n+K_n}$*, where* $\mathbf{I}_{J_n+K_n}$ *is the identity matrix of order* $J_n + K_n$*.*

Under these definitions, the private information received by investor n consists of the realization of the random vector $\mathbf{M}_n\tilde{\eta}$. As is usual in a REE framework, equilibrium prices also reveal some information to investors beyond their own private information. Consequently, each investor n maximizes his expected utility of consumption conditional on the realization of his private information and on the observation of the public information in the form of prices at date 0.

2.2 Equilibrium Prices and Holdings

We seek solutions for prices and holdings at date 0 within the class of functions that are linear in our information variable $\tilde{\eta}$ and supply variable $\tilde{z}$. The form of the solution implies investors assume prices are a linear function of private signals and noise. In equilibrium, this hypothesis is verified. The date 0 price vector is:

$$\tilde{P}^0 = A_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z} \tag{4}$$

where A_0 is a $J \times 1$ vector, $\mathbf{A}_1$ is a $J \times (J + K)$ matrix, and $\mathbf{A}_2$ is a $J \times J$ matrix. We suppose that $\mathbf{A}_2$ is regular. Under these assumptions, investor n's demand is:

$$\tilde{X}_n = a^{-1}\mathbf{V}_n^{-1}\left(E_n\left[\tilde{P}^1\right] - R\tilde{P}^0\right) \tag{5}$$

Equation (5) gives an expression for agent n's holdings at date 0—please see Appendix B for additional details. All appendices are in the Internet Appendix and can be accessed by the URL on the front page of this document. The expression $E_n[\tilde{P}^1] = E[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$ gives the expected prices of the risky assets at date 1 from investor n's point of view (i.e. conditional on his information set). $\mathbf{V}_n = Var[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$ represents the conditional variance of $\tilde{P}^1$ from investors n's point of view. By equating the supply and the aggregate demand of the N groups of investors, $\left(\sum_{n=1}^{N} \lambda_n \tilde{X}_n = \tilde{z}\right)$, it follows:

$$\sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left(E_n\left[\tilde{P}^1\right] - R\tilde{P}^0\right) - a\tilde{z} = 0 \tag{6}$$

Joint normality implies that the distribution of prices, conditional on investor n's private and public information, is also multi-variate normal with the following expectation:

$$\begin{aligned} E_n\left[\tilde{P}^1\right] &= E\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] \\ &= B_{0n} + \mathbf{B}_{1n}\mathbf{M}_n\tilde{\eta} + \mathbf{B}_{2n}\tilde{P}^0 \end{aligned} \tag{7}$$

where the dimension of B_{0n} is $J \times 1$, $\mathbf{B}_{1n}$ is $J \times (J_n + K_n)$, and $\mathbf{B}_{2n}$ is $J \times J$. Equations (4), (6), and (7) imply the system to be solved is (please see Appendix C):

$$\begin{aligned} a\mathbf{A}_2^{-1}A_0 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} B_{0n} \\ a\mathbf{A}_2^{-1}\mathbf{A}_1 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \mathbf{B}_{1n}\mathbf{M}_n \\ a\mathbf{A}_2^{-1} &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left(R\mathbf{I}_J - \mathbf{B}_{2n}\right) \end{aligned} \tag{8}$$

To conclude this section, Equation (4) in conjunction with the equations in (8) give equilibrium prices. Equation (5) gives equilibrium holdings of investors in group n. As shown in Appendix C, the matrices $\mathbf{B}_{1n}$, $\mathbf{B}_{2n}$ and $\mathbf{V}_n$ can be written as functions of the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$. The system of equations in (8) represents a fixed point problem in a $2J^2 + JK + J$ Euclidian space. Such a system can be solved numerically for small values of J and K.

3 Numerical Analysis

This section numerically analyzes relations between information structures and equilibrium asset prices. We vary which investor group has which pieces of information while leaving the model parameters $\left\{r_f, a, \lambda_1, \ldots, \lambda_N, \overline{\theta}, \mathbf{B}, \mathbf{\Sigma}_\varepsilon, \mathbf{\Sigma}_\theta, \mathbf{\Sigma}_z\right\}$ constant. Rather than have conclusions depend on realizations of random variables, we study "ex-ante prices" by taking expectations over $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$, the three random variables in the model.[7]

We consider four non-overlapping groups of investors, each with equal numbers, and denoted A, B, C, and D. There are four assets denoted 1, 2, 3, and 4. Finally, there are two common-components denoted f_1 and f_2. We use two common components for parsimony and to differentiate from models that have a single global factor. In reality, our model allows for multiple common components and we could have shown structures with three or more factors (and five or more assets). We note that different information structures correspond to investor groups having different combinations of asset-specific and/or common-component information.

[Insert Table 2 About Here]

Table 2 presents 26 different information structures. The first structure represents a frictionless world in which each investor group knows all possible asset-specific and common-component information. To read the table, on the full-info CAPM line Group A has asset-specific information about assets 1, 2, 3, and 4 as well as information about common components f_1 and f_2. Figure 1, Panel A graphically depicts this symmetric and complete information structure (i.e., the frictionless/full-information CAPM world).

Below the full-info CAPM, Table 2's next ten lines show other symmetric structures. Each investor group has the same information and no group has complete information. Below the symmetric cases, there are other information structures that are inspired by recently

[7] As noted, our approach involves taking expectations over random variables. An alternative methodology involves drawing a set of random variables $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$ and calculating prices and holdings at date 0. Repeated draws of the random variables converge to the same expected values as the number of draws goes to infinity. Note that our methodology solves for prices before agents receive private information. As such, solutions are sometimes referred to as *ex-ante*. Appendix F shows how one solves for closed-form, date 0, ex-ante prices using the full-info CAPM structure as an example.

published papers. In Albuquerque, Bauer, and Schneider (2009) there is a single global factor (called f_1 in our paper and G in their paper) which is known by one group of investors. From Table 2, the base case version assumes group D knows the single global factor. We also consider three other structures in the spirit of Albuquerque, Bauer, and Schneider (2009) and called $v1$, $v2$, and $v3$. In these structures, there are two global factors. In structures $v1$ and $v2$, group D has information about both global factors. Internet Appendix G graphically depicts the Albuquerque, Bauer, and Schneider (2009) base case information structure along with structure $v2$.

Kodres and Pritsker (2002) allow asset prices to be determined by an underlying factor structure, but no group of investors has information about the factors. We model three structures consistent with the Kodres and Pritsker (2002) paper—the first two of which are shown in Internet Appendix G.

Finally, we consider eight additional structures. In the structure labeled Additional $v1$, Groups B, C and D have exactly the same information sets. In the structure labeled Additional $v3$, the amount of information increases from Groups A to B to C to D. Group D has asset-specific information for assets 1, 2, 3, and 4 as well as information about both f_1 and f_2. Internet Appendix G shows the second and third of these eight additional structures. Figure 1, Panel B shows the Additional v6 structure.

The model's parameters are kept simple in the numerical analysis so as to focus on relations between prices and information structures. The expected value of $\tilde{\theta}$ is 20 for all assets and $\mathbf{\Sigma}_\theta$ is the identity matrix. The riskfree rate is zero. Investors are evenly distributed across groups ($\lambda_n = 0.25$ for n=1, ..., 4.). The expected value of $\tilde{z}$ is one for all assets and $\mathbf{\Sigma}_z$ is the identity matrix. These assumptions imply there are four shares of each asset in expectation. All factor loadings in $\mathbf{B}$ are one and the risk aversion coefficient is one.

[Insert Table 3 About Here]

To efficiently display results from the numerical analysis, Table 3 reports the aggregate market capitalization associated with each of the 26 structures. The aggregate market cap is the sum of prices for assets 1, 2, 3, and 4. The full-info CAPM world, not surprisingly, results in the highest prices and has a total market capitalization of \$304.00. As information frictions become more severe, payoffs become relatively more risky, and prices fall. In the structure "Symmetric v10" each investor group has asset-specific information about only one asset and no group has common-component information. The total market capitalization is \$164.00 which is \$140.00 below the full-info CAPM value. The price discount is 46% as indicated in the third column and calculated as \$140.00 ÷ \$304.00. Note that the proportional discount from the full-info CAPM-price levels represents one way to measure aggregate informational frictions in a market. Table 3 shows the "degree of frictions" ranges from 0% to 46%.

Although not shown in Table 3, we note there is also a symmetric and frictionless CAPM in which no group of investors has any private information. We refer to such a structure as the "No-Information CAPM". This structure produces a \$160.00 total market capitalization which corresponds to a 47% price discount. Either the full-info CAPM or the no-info CAPM can serve as a baseline by which to measure the effects of different information structures on equilibrium prices. If the latter is used, additional bits of information lead to higher and higher prices and Table 3 can be read from bottom to top.

4 Closed-Form Solutions for Prices and Holdings

To get closed-form solutions for prices and holdings, we make certain (rather weak) assumptions about which group has which information. This section first describes the informational assumptions. Second, we present the closed-form solutions. Third, we analyze equilibrium expressions for prices and holdings. This subsection provides economic insights available from the solutions. Fourth, we evaluate the restrictiveness of the assumptions in Section 4.1. We conclude the structures are not restrictive and can model a wide range of information structures and markets.

4.1 Assumptions About Information Structures

Asset-Specific Information: The J securities are partitioned into N non-overlapping groups.[8] We define the set of all assets as S. The set of assets in group n contains J_n risky assets and is denoted S_n. Thus, $\bigcup_{n=1}^{N} S_n = S$ and $\forall(n_a, n_b)$, $n_a \neq n_b$, $S_{n_a} \cap S_{n_b} = \oslash$.

A single investor i in group n knows the realization of the asset-specific component, θ_j, of each asset j in the set S_n. For any asset j not in S_n, investor i only knows the distribution of $\tilde{\theta}_j$ but he does not know its realization. We assume there is an equal number (N) of securities groups and investors groups to ensure that each security has at least one investor with asset-specific information.

Common Component Information: We assign each of the K common factors to one of N groups denoted F_n, with $n = 1, \ldots, N$. The set F_n contains K_n common components and $0 \leq K_n \leq K$. An investor i in group n knows the realization of each common component $\tilde{f}_k$ in the set F_n. For any component not in F_n, the investor only knows the distribution of $\tilde{f}_k$ but not its realization. For tractability purposes of the model, we assume that two groups of investors do not have information about the same common component.[9]

4.2 Closed-Form Solutions for Prices and Holdings

To obtain a closed-form solution for $\tilde{P}^0$, we define the matrix $\mathbf{U} \equiv \mathbf{A}_2^{-1}\mathbf{A}_1$. We also introduce the function $g(\mathbf{G}) = \sum_{n=1}^{N} \mathbf{D}_n \mathbf{G} \mathbf{D}_n$, where $\mathbf{G}$ is a matrix of order $J + K$. The function $g(\cdot)$ transforms a matrix $\mathbf{G}$ into a N-block diagonal matrix whose block elements are the same as the elements of the matrix $\mathbf{G}$.

> **Definition 4.** *We define a "g-matrix" to be any square matrix* $\mathbf{G}$ *of order* $J + K$ *which satisfies* $g(\mathbf{G}) = \mathbf{G}$. *This means that* $\mathbf{G}$ *is an* N*-block diagonal matrix, the size of block* n *is equal to the number of specific and common components known by investor* n.

[8] Note that Section 2.1 has already partitioned investors into N non-overlapping groups.

[9] Chen, Roll, and Ross (1986) document nine macroeconomic risk factors affecting stock returns. We likewise envisage the number of common components to be much less than the number of assets, $K \ll J$. If the number of common components is less than the number of investor groups, then $K < N$, some F_n sets will not contain any common components ($K_n = 0$), and the corresponding investor group will not be informed about any of the common components. Note that $K < N$ in Figure 1.

Define $\mathbf{\Psi} \equiv Var\left[\tilde{\eta}|\tilde{P}^0\right]$ i.e., the variance-covariance matrix of $\tilde{\eta}$ conditional on observing the equilibrium price vector at date 0. The matrix $\mathbf{\Psi}$ is endogenously defined and represents the variance of $\tilde{\eta}$ from the point of view of an investor who does not possess any private information but only observes the equilibrium price vector. The following lemma gives an analytical solution for $\mathbf{U}$.

Lemma 1. *If $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C})$ is a g-matrix, then the closed-form solution for $\mathbf{U}$ is:*

$$\mathbf{U} = a^{-1}\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D} \tag{9}$$

Proof: *See Appendix D.*

For the particular case of Lemma (1), $\mathbf{U}$ is not a function of the coefficients B_{0n}, $\mathbf{B}_{1n}$, and $\mathbf{B}_{2n}$. Therefore, to determine A_0, $\mathbf{A}_1$, and $\mathbf{A}_2$, we must first compute the matrix $\mathbf{\Psi}$ as a function of $\mathbf{U}$. In this way, the variance-covariance matrix of any investor group, $\mathbf{V}_n$, can be written as a function of $\mathbf{\Psi}$:

$$\mathbf{V}_n \quad = \quad \mathbf{\Sigma}_\varepsilon + \mathbf{C}\mathbf{\Psi}\mathbf{C}' - \mathbf{C}\mathbf{\Psi}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi}\mathbf{C}' \tag{10}$$

Where $\mathbf{\Psi}_n = \mathbf{M}_n\mathbf{\Psi}\mathbf{M}_n'$. Also, $\mathbf{\Psi} = \mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}$ and $\mathbf{M} = \mathbf{U}\mathbf{Q}\mathbf{U}' + \mathbf{\Sigma}_z$. The following theorem gives a closed-form solution for the equilibrium price vector at date 0.

Theorem 1. *Under the conditions of Lemma (1), there exists a closed-form solution for Equation (6) within the class of linear functions of $\tilde{\eta}$ and $\tilde{z}$. The solution can be written as, $\tilde{P}^0 = A_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$, where $\mathbf{A}_2$ is a regular matrix and:*

$$A_0 \quad = \quad \frac{1}{R}\Big(\big(\mathbf{C} - R\mathbf{A}_1\big)E[\tilde{\eta}] + \big(R\mathbf{A}_2 - a\mathbf{V}_N\big)E[\tilde{z}]\Big) \tag{11}$$

$$\mathbf{A}_1 \quad = \quad \frac{1}{R}\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_\varepsilon - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\mathbf{C}\mathbf{D} \tag{12}$$

$$\mathbf{A}_2 \quad = \quad \frac{1}{R}a\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma}_\varepsilon - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\mathbf{\Sigma}_\varepsilon \tag{13}$$

The matrix $\mathbf{V}_N = (\sum_{n=1}^N \lambda_n \mathbf{V}_n^{-1})^{-1}$ represents the variance-covariance matrix of $\tilde{P}^1$ for the "average" investor in the market. The precision matrix $\mathbf{V}_N^{-1}$ equals the weighted mean of each group's precisions where the weights are proportional to the number of agents in each group. From Equation (10), it is straightforward *to show that* $\mathbf{V}_N$ *can be written as:*

$$\mathbf{V}_N = (\mathbf{\Sigma}_\varepsilon + \mathbf{C\Psi C}')(\mathbf{I}_J + \mathbf{\Sigma}_\varepsilon^{-1}\mathbf{CD\Psi C}')^{-1} \tag{14}$$

Proof: *See Appendix E.*

To conclude, we provide closed-form solutions for prices and holdings at date 0. The solution for prices takes the form shown in Equation (4) with constant values shown in (11), (12), and (13). The solution for investor group n's holdings is given by Equation (5).

4.3 Analysis of Equilibrium Asset Prices

We analytically analyze relations between information structures and equilibrium asset prices. To do this, we vary structures while leaving the model parameters constant. This approach is the closed-form version of the numerical "ex-ante" pricing mentioned in Footnote 7.

General Model with Disperse Information: Rearranging Equation (6) gives a general expression for prices at date 0. Equation (15) below shows that asset prices at date 0 are less than the value of expected future payoffs.[10] The total price discount (risk premium) is given by the expression $a\mathbf{V}_N E[\tilde{z}]$. The price discount depends on risk aversion (a) and the market's "average" uncertainty about future payoffs ($\mathbf{V}_N$).

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{V}_N E[\tilde{z}]\right) \tag{15}$$

Model with Symmetric and Complete Information: When all investors are informed about all asset-specific components and common components, our equations reduce to a form of the full-information Capital Asset Pricing Model (or full-info CAPM), expressed in term of prices, and adjusted for supply uncertainty. See Appendix F for details of related calculations. The appendix also shows the full-info CAPM adjusted for supply uncertainty

[10]Assuming assets are expected to be in positive net supply ($E[\tilde{z}] > 0$) and agents are risk averse ($a > 0$).

and expressed with covariance terms—a form that is more familiar to financial economists.

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{\Sigma}_\varepsilon E\left[\tilde{z}\right]\right) \tag{16}$$

Information Discount Factor: We define the "information discount factor" (or DF_{info}) as the difference between the price discounts shown in Equations (15) and (16). The DF_{info} represents the amount an asset's price at date 0 is below its expected future value due to agents not having full information about future payoffs.

$$\begin{aligned} DF_{info} &\equiv \frac{a}{R}\mathbf{V}_N E\left[\tilde{z}\right] - \frac{a}{R}\mathbf{\Sigma}_\varepsilon E\left[\tilde{z}\right] \qquad (17) \\ &= \frac{a}{R}(\mathbf{V}_N - \mathbf{\Sigma}_\varepsilon)E\left[\tilde{z}\right] \end{aligned}$$

In a single-asset model with no factor structure, the information discount factor is proportional to the difference between the market's average uncertainty about future payoffs ($\mathbf{V_N}$) and residual uncertainty about the same payoffs ($\mathbf{\Sigma}_\varepsilon$). This difference is a signal-to-noise measure. When the difference is small, investors have a lot of information about future payoffs, the DF_{info} is low, and prices are high. Note that $DF_{info} \geq 0$ as the market is always bounded in its assessment of future payoffs by $\mathbf{\Sigma}_\varepsilon$.

In a multi-asset model with uncorrelated residual uncertainties and no factor structure, the single-asset intuition discussed in the paragraph above continues to hold. The diagonal matrix $(\mathbf{V}_N - \mathbf{\Sigma}_\varepsilon)$ represents a series of signal-to-noise differences.

In a multi-asset model with correlated residual uncertainties and/or a factor structure, the information discount factor can be driven by both the asset-specific components of payoffs and common factors. The matrix $(\mathbf{V}_N - \mathbf{\Sigma}_\varepsilon)$ can still be roughly interpreted as signal-to-noise differences. However, the matrix is no longer diagonal which means that covariance terms affect the DF_{info}. Section 3 of this paper numerically analyzes asset prices in an effort to better understand the role of the covariance terms.

4.4 Analysis of Investor Holdings (Portfolio Choice)

We analytically analyze relations between information structures and investor group n's holdings of risky assets. We vary structures, leave the model parameters constant, and measure ex-ante holdings. To do this, we take expectations of Equations (5) and (6) and rearrange terms to give:

$$\begin{aligned} E\left[X_n\right] &= a^{-1}\mathbf{V}_n^{-1}\left(E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right]\right) \\ &= \mathbf{V}_n^{-1}\mathbf{V}_N E\left[\tilde{z}\right] \end{aligned} \tag{18}$$

In a single-stock world with no common components, investor n's holdings depends on the ratio of the market's uncertainty about the future payoff ($\mathbf{V}_N$) to his own uncertainty about the same payoff ($\mathbf{V}_n$). The higher the investor's uncertainty relative to the market, the lower the ratio, and the lower the weight of the asset in his portfolio.

In a multi-asset framework with uncorrelated residual uncertainty and no common components, the matrices ($\mathbf{V}_N$) and ($\mathbf{V}_n$) are diagonal. The term $\mathbf{V}_n^{-1}\mathbf{V}_N$ represents a series of uncertainty ratios. The same intuition described in the paragraph above holds.

In a multi-asset model with correlated residual uncertainties and/or a factor structure of payoffs, thinking about $\mathbf{V}_n^{-1}\mathbf{V}_N$ as a ratio of two uncertainty measures provides rough intuition only. However, the ratio of two matrices includes covariance terms relating to uncertainty about assets' payoffs. Investor n's holdings of a specific asset now depends on his uncertainty about the asset's payoffs, his uncertainty about other assets' payoffs, and other investors' uncertainty about all assets (including the asset in question). These uncertainties can arise from information asymmetries about the asset-specific components of payoffs and/or common components.

4.5 Restrictiveness of Assumptions

We use two methods to evaluate restrictiveness of the assumptions in Section 4.1. The first method uses the aggregate market capitalizations shown in Table 3. We plot the market capitalization as a function of the degree of friction in the economy (by definition, these quantities define a straight line). Figure 2 shows the plot.

[Insert Figure 2 About Here]

Of the 26 information structures, we obtain closed-form solutions for 22 of them and these are marked by blue circles on the line. Notice that the red diamonds are in the "interior" of the line segment. That is, the structures for which we cannot generate closed-form solutions are not related to extreme price discounts. Similar price discounts can be obtained in closed-form by using a different information structure.

[Insert Figure 3 About Here]

The second method introduces a measure of aggregate holdings dispersion. For each investor-asset combination, we compare the portfolio weight to the full-info CAPM weight for the same asset. We then calculate the root mean squared error (RMSE) across all investor-asset combinations. The RMSE provides a single aggregate measure that allows us to quantify how disperse holdings are relative to the full-info CAPM. Figure 3 shows a plot of holdings dispersion as a function of the degree of frictions in the economy.

To generate additional points in Figure 3, we consider three values for the main diagonal elements of $\mathbf{\Sigma}_\theta$ and three values for all elements of $\mathbf{B}$. Varying $\mathbf{\Sigma}_\theta$ affects the value of asset-specific information. Varying $\mathbf{B}$ affects the value of common component information. Note that neither parameter affects the aggregate market capitalization in the full-info CAPM world as the amount of residual uncertainty ($\mathbf{\Sigma}_\epsilon$) is left unchanged. In total, we generate $3\times3\times26 = 234$ possible points.[11] Table 4 shows the aggregate holdings dispersion measure (RMSE) for each of the 26 information structures and for three of the nine parameter combinations.

[Insert Table 4 About Here]

[11] Parameter combinations are virtually unlimited in our model. Here, we choose simple permutations so as to build economic intuition. We set all the main diagonal elements of $\mathbf{\Sigma}_\theta$ to 1.0 or 2.5 or 5.0. Also, all elements of $\mathbf{B}$ are set to 0.5, or all are set to 1.0, or all are set to 1.5. In actuality, our model is much more general and elements in $\mathbf{B}$ can vary across assets and factors. Here, we again want to build economic intuition by keeping parameters simple.

Figure 3 shows a "cloud" of different dispersion measures. All symmetric structures have zero dispersion and can be found on the X-axis. We again plot those structures that meet our closed-form assumptions with blue circles. Notice that the red diamonds again are in the "interior" of the cloud. We conclude that structures capable of generating the most disperse holdings meet our closed-form solutions. The highest level of dispersion is associated with Albuquerque, Bauer, and Schneider (2009) $v2$ and parameters $\mathbf{\Sigma}_\theta = 5$, $\mathbf{B}{=}\mathbf{1.5}$. The RMSE of this structure is 0.38. An alternative measure of holdings dispersion is mean absolute deviation from the full-info CAPM weights (or "MAD"). Although not reported, the MAD is 0.28. Looking at one information structure previously discussed in this paper, "Additional $v6$" has frictions of 0.31 on the X-axis and a 0.08 RMSE on the Y-axis.

5 Conclusion

This paper proposes a multi-asset, rational expectations equilibrium model in which agents are asymmetrically informed about asset-specific and common components of payoffs. Our model allows agents to have asset-specific information and/or common component information. The model produces closed-form solutions for asset prices as well as for the holdings of individual agents.

Our solution for equilibrium prices is general and can be applied to numerous information structures. We solve the model for the case when all investors have symmetric and complete information. We solve for other cases when investors are asymmetrically informed and/or do not have complete information. Our analysis leads to a closed-form solution for the information discount factor (or DF_{info}) which is the amount equilibrium prices are reduced due to agents not having full information about assets' future payoffs. The DF_{info} can be used to quantify the degree of informational frictions in the economy. A higher degree of informational frictions leads to a higher DF_{info} and lower prices.

The first paragraph of this paper asks: How do market prices differ when one group of investors has all common-component information compared with situations when different groups have information about different common components? We now have an answer—

aggregate prices in the first structure are 22% higher than in the second structure.[12] The general form of our solutions allow us to ask and answer a host of additional questions. For example, how do prices change if the two groups without common-component information share their information?[13] We can even ask: For a given structure, how do prices vary as factor loadings change? Expanding Table 4 to include market capitalizations would quickly show answers to this question. Finally, and for a given a set of parameters, the ability to model different structures allows us to say something about the impact of asset-specific information vs. common-component information.

Why might some 401(k) plans invest heavily in their own company's stock? Consider information structures such as those shown in Figure 1 where employees are partitioned into four groups, there are four companies, and two common components. If investors feel they have superior asset-specific information, they may want to increase the weight of their own company's stock. However, investors will also consider how important economy-wide factors are to the stock. If factor loadings are large in absolute magnitudes and if others are likely to have valuable information about the factors, investors will decrease the weight of the stock.

Although not the exclusive focus of our paper, we can also discuss the question: Why do international mutual funds invest heavily in some foreign equities but not in others? In a simple framework such as in Gehrig (1993), investors are generally assumed to have information about their home county's assets (or view the assets as less risky). The last figure in Appendix G depicts such an information structure. Such a structure generates home bias indicating an investor holds more of an asset than he would if he invested in the world market portfolio. However, generating large levels of information dispersion is difficult in these structures. Generally, investors overweight their own country's stocks and (equally) underweight all other country's stocks.

In addition to home bias, our model can generate reverse (or negative) home bias indicating an investor underweights his home country's assets relative to world market portfolio weights—see Bravo-Ortega (2003). A strength of our model is its ability to produce large

[12]From Tables 2 and 3, the structure called "Albuquerque et al. v2" endows Group D with all common-component information and has a market capitalization of \$255.00. The structure "Alternative v8" endows Group C with information about f_1 and Group D with information about f_2 and has a market capitalization of \$208.18.

[13]The structure "Additional v6" produces answers to this question and a market capitalization of \$209.53.

variations in home bias as well as its ability to produce reverse home bias. The intuition comes from thinking about economy-wide information. If common components play a large role in determining an asset's payoff, those with information about the components are likely to overweight the asset regardless of whether it comes from one's home country or from a foreign country.

Studies related to home bias can/do focus on areas beyond international portfolio choice. Examples include individual ownership of own-company stock (such as the 401(k) example), ownership patterns determined by investors' job locations/industries, ownership patterns determined by stocks' industries, and intra-national home bias as in Coval and Moskowitz (1999). All of these examples have an inherent tension between the value of company-specific information and the value of common-component information. Our model provides insights into all situations. For example, and given the right data, one could test whether stocks with high levels of ownership (by investors working in the same industry) load less significantly on any industry-wide factors.

There are a number of additional avenues for potential future research. First, one could try to extend our model to multiple periods. This would provide expressions for net trading as in Brennan and Cao (1997) as well as suggest empirical tests based on trading (as opposed to holdings) data. Second, one could work to devise methods of empirically identifying different information structures. While no small task, structures could then be used to test relative asset prices using expressions in this paper. Third, our model may be adapted to better understanding partially segmented markets. In such cases, information reflects the "friction" that segments markets. One may be able to model groups of investors who face low frictions only when trading securities from their home country, groups of investors who face low frictions when trading securities in a contiguous block of countries (a geographic region), or groups of investors who face low frictions when investing in any global security. None of the three extensions is likely to be easy—all are potentially interesting.

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Table 1: Variable Descriptions and Definitions

a	The coefficient of constant absolute risk aversion.
$\tilde{f}$	Vector of K common components with $\tilde{f} = (\tilde{f}_1 \dots \tilde{f}_K)'$.
r_f	Riskfree rate with $R = (1 + r_f)$
$\tilde{w}_i^t$	Wealth of investor i and date t.
$\tilde{z}$	Vector of per capital asset supplies with $\tilde{z} = (\tilde{z}_1 \dots \tilde{z}_K)'$.
————	————
$\tilde{\epsilon}$	Vector of residual uncertainties with $\tilde{\epsilon} = (\tilde{\epsilon}_1 \dots \tilde{\epsilon}_J)'$.
$\tilde{\eta}$	The "stacked" vector of asset-specific and common component with $\tilde{\eta} = (\tilde{\theta}' \tilde{f}')'$
λ_n	The fraction of total investors (I) in group n such that $\sum_{n=1}^{N} \lambda_n = 1$.
$\tilde{\theta}$	Vector of assets specific component of payoffs with $\tilde{\theta} = (\tilde{\theta}_1 \dots \tilde{\theta}_J)'$.
————	————
$\mathbf{\Psi}$	The variance-covariance matrix of $\tilde{\eta}$ conditional on observing the equilibrium price vector at date 0.
$\mathbf{\Sigma}_\epsilon$	The covariance matrix of the residual uncertainty.
$\mathbf{\Sigma}_\theta$	The covariance matrix of the asset-specific component of payoffs.
$\mathbf{\Sigma}_z$	The covariance matrix of the supply shocks.

A_0, $\mathbf{A}_1$, $\mathbf{A}_2$	One vector and two matrices of constants in the price equation.
B_{0n}, $\mathbf{B}_{1n}$, $\mathbf{B}_{2n}$	One vector and two matrices of constants in the price equation.
$\mathbf{B}$	$J \times K$ matrix of factor loadings.
$\mathbf{C}$	$J \times (J+K)$ block-diagonal matrix consisting of $\mathbf{I}_J$ and $\mathbf{B}$.
$\mathbf{D}$	Defined as $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$.
$\mathbf{D}_n$	$\mathbf{D}_n$ is a diagonal matrix of order $J+K$ with ones on the main diagonal if investors in group n know an asset's or a factor's information.
$\mathbf{G}$	Square matrix of order $J+K$ such that $g(\mathbf{G}) = \sum_{n=1}^{N} \mathbf{D}_n.\mathbf{G}\mathbf{D}_n = \mathbf{G}$
I	Total number of investors
$\mathbf{I}_J$	A $J \times J$ identity matrix.
$\mathbf{I}_K$	A $K \times K$ identity matrix.
J	Number of assets.
J_n	Number of assets for which investors in group n have asset-specific information.
K	Number of common components (factors) in the economy.
K_n	Number of common components for which investors in group n have information.
$\mathbf{M}_n$	This matrix is obtained by eliminating the null (zero) rows of $\mathbf{D}_n$.
N	Number of non-overlapping groups
$\tilde{P}^0$	Vector of equilibrium prices at date 0 with $\tilde{P}^0 = (\tilde{P}_1^0 \ldots \tilde{P}_J^0)'$
$\tilde{P}^1$	Vector of final payoffs with $\tilde{P}^1 = (\tilde{P}_1^1 \ldots \tilde{P}_J^1)'$
$\mathbf{Q}$	The variance-covariance matrix of $\tilde{\eta}$.
R	Gross riskfree rate with $R = (1 + r_f)$
$\mathbf{U}$	Matrix defined as: $\mathbf{U} \equiv \mathbf{A}_2^{-1}\mathbf{A}_1$.
$\mathbf{V}_N$	Conditional variance of $\tilde{P}^1$ from "average" investor's point of view.
$\mathbf{V}_n$	Conditional variance of $\tilde{P}^1$ from investor n's point of view.
$\tilde{X}_n$	The holdings of investor group n.

Table 2: Summary of Different Information Structures

The table overview different information structures studied in our numerical analysis. We consider four groups of investors labeled A, B, C, and D, four assets numbered 1, 2, 3, and 4, and two factors f_1 and f_2. For each group of investors and each structure, we use asset numbers to note whether the investor group has asset-specific information. We use the factor number to note whether the investor group has factor information. We model information structures consistent with those studied in the frictionless/full-information CAPM, symmetric structures, Albuquerque, Bauer, and Schneider (2009), Kodres and Pritsker (2002), as well as some additional information structures.

	Investor A	Investor B	Investor C	Investor D
Full-Info CAPM	1, 2, 3, 4, f_1, f_2	1, 2, 3, 4, f_1, f_2	1, 2, 3, 4, f_1, f_2	1, 2, 3, 4, f_1, f_2
Symmetric v1	1, 2, 3, f_1, f_2	1, 2, 3, f_1, f_2	1, 2, 3, f_1, f_2	1, 2, 3, f_1, f_2
Symmetric v2	1, 2, f_1, f_2	1, 2, f_1, f_2	1, 2, f_1, f_2	1, 2, f_1, f_2
Symmetric v3	f_1, f_2	f_1, f_2	f_1, f_2	f_1, f_2
Symmetric v4	1, 2, 3, f_1	1, 2, 3, f_1	1, 2, 3, f_1	1, 2, 3, f_1
Symmetric v5	1, 2, f_1	1, 2, f_1	1, 2, f_1	1, 2, f_1
Symmetric v6	1, f_1	1, f_1	1, f_1	1, f_1
Symmetric v7	f_1	f_1	f_1	f_1
Symmetric v8	1, 2, 3	1, 2, 3	1, 2, 3	1, 2, 3
Symmetric v9	1, 2	1, 2	1, 2	1, 2
Symmetric v10	1	1	1	1
Albuquerque et al. (base)	1	2	3	4, f_1
Albuquerque et al. v1	1, 2, 3	1, 2, 3	1, 2, 3	4, f_1, f_2
Albuquerque et al. v2	1	2	3	4, f_1, f_2
Albuquerque et al. v3	1, 2, 3, f_1	1, 2, 3, f_1	1, 2, 3, f_1	4, f_2
Kodres & Pritsker (base)	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4
Kodres & Pritsker v1	1, 2	2, 3	3, 4	4, 1
Kodres & Pritsker v2	1	2	3	4
Additional v1	1	2, 3, 4, f_1, f_2	2, 3, 4, f_1, f_2	2, 3, 4, f_1, f_2
Additional v2	1	2	3, 4, f_1, f_2	3, 4, f_1, f_2
Additional v3	1	1, 2, f_1	1, 2, 3, f_2	1, 2, 3, 4, f_1, f_2
Additional v4	1	2	3, f_1, f_2	4, f_1, f_2
Additional v5	1, f_1, f_2	1, 2, f_1	1, 2, 3, f_2	1, 2, 3, 4
Additional v6	1, 2	1, 2	3, f_1	4, f_2
Additional v7	1, f_1	2, f_2	3, 4	3, 4
Additional v8	1	2	3, f_1	4, f_2

Table 3: Aggregate Market Capitalizations

This table summarizes aggregate market capitalizations (”MC”) of different information structures. The ”Price Discount from CAPM” represents the difference between prices in a given structure and prices in the frictionless CAPM world. The quantity ”Degree of Frictions” is defined as the ratio of the price discount to the market capitalization of the CAPM world. Structures are listed in order of decreasing market capitalization (i.e., increasing degree of frictions).

	MC ($)	Price Discount from CAPM	Degree of Frictions
Full-Info CAPM	304.00	0.00	0.00
Symmetric v1	300.00	4.00	0.01
Additional v1	297.55	6.45	0.02
Symmetric v2	296.00	8.00	0.03
Symmetric v3	288.00	16.00	0.05
Additional v2	285.04	18.96	0.06
Additional v3	281.27	22.73	0.07
Additional v4	279.60	24.40	0.08
Additional v5	266.64	37.36	0.12
Albuquerque et al. v1	257.35	46.65	0.15
Albuquerque et al. v2	255.00	49.00	0.16
Albuquerque et al. v3	238.16	68.00	0.22
Symmetric v4	236.00	68.00	0.22
Symmetric v5	232.00	72.00	0.24
Symmetric v6	228.00	76.00	0.25
Symmetric v7	224.00	80.00	0.26
Additional v6	209.53	94.47	0.31
Additional v7	209.53	94.47	0.31
Additional v8	208.18	95.82	0.32
Albuquerque et al. (base)	190.12	113.88	0.37
Kodres & Pritsker (base)	176.00	128.00	0.42
Symmetric v8	172.00	132.00	0.43
Kodres & Pritsker v1	170.98	133.02	0.44
Symmetric v9	168.00	136.00	0.45
Kodres & Pritsker v2	166.49	137.51	0.45
Symmetric v10	164.00	140.00	0.46

Table 4: Closed-Form Solutions and Holdings Dispersion

This table summarizes dispersion across investors' holdings that are associated with different information structures. The first column gives the name of 26 information structures considered. The second column shows the 22 structures that meet Section 4.1's assumptions needed for closed-form solutions. The last three columns show holdings dispersion when parameter $\mathbf{\Sigma}_\theta = 1.0$ and $\mathbf{B} = 0.5$, 1.0, or 1.5. We compare portfolio weights to the full-info CAPM weights. Holdings dispersion is defined as the root mean squared error (RMSE) across all investor-asset combinations.

	Meets Closed-Form Assumptions	<——— Holdings Dispersion		———>
		$\mathbf{\Sigma}_\theta$1.0 **B**0.5	$\mathbf{\Sigma}_\theta$1.0 **B**1.0	$\mathbf{\Sigma}_\theta$1.0 **B**1.5
Full-Info CAPM	Yes	0.000	0.000	0.000
Symmetric v1	Yes	0.000	0.000	0.000
Symmetric v2	Yes	0.000	0.000	0.000
Symmetric v3	Yes	0.000	0.000	0.000
Symmetric v4	Yes	0.000	0.000	0.000
Symmetric v5	Yes	0.000	0.000	0.000
Symmetric v6	Yes	0.000	0.000	0.000
Symmetric v7	Yes	0.000	0.000	0.000
Symmetric v8	Yes	0.000	0.000	0.000
Symmetric v9	Yes	0.000	0.000	0.000
Symmetric v10	Yes	0.000	0.000	0.000
Albuquerque et al. (base)	Yes	0.085	0.088	0.090
Albuquerque et al. v1	Yes	0.060	0.094	0.144
Albuquerque et al. v2	Yes	0.088	0.120	0.168
Albuquerque et al. v3	Yes	0.058	0.060	0.061
Kodres & Pritsker (base)	Yes	0.000	0.000	0.000
Kodres & Pritsker v1	No	0.077	0.079	0.079
Kodres & Pritsker v2	Yes	0.085	0.086	0.086
Additional v1	Yes	0.063	0.068	0.070
Additional v2	Yes	0.088	0.110	0.127
Additional v3	No	0.033	0.026	0.021
Additional v4	No	0.863	0.102	0.116
Additional v5	No	0.058	0.091	0.135
Additional v7	Yes	0.078	0.081	0.084
Additional v6	Yes	0.078	0.081	0.084
Additional v8	Yes	0.085	0.088	0.090

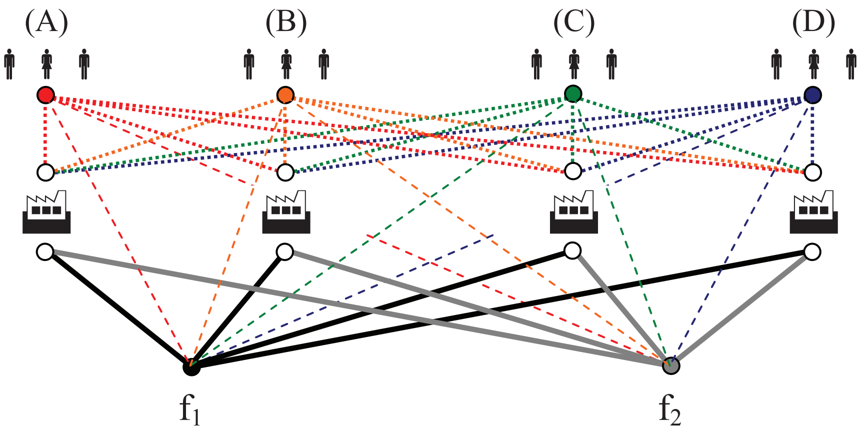
Figure 1

Two Possible Information Structures

These diagrams depict worlds with four groups of investors labeled A, B, C, and D. There are four assets depicted by factories and two factors denoted "f_1" and "f_2". Solid lines from factors to assets indicate that payouts are determined by an underlying factor structure. Dashed lines from investor groups to either assets or to factors indicate investor groups possess information on these assets or factors.

Panel A: Symmetric and Complete Information (Frictionless or Full-Info CAPM)

The diagram depicts the information structure in a full-info CAPM world. In this diagram, each group of investors has information about each asset and each factor. No investor has an information advantage or disadvantage. All possible information about each asset's payoff (other than the residual uncertainty) is known.



Panel B: Additional Structure v6

The diagram depicts information structure that is denoted "Additional v6" in the tables. Investors group A and group B have asset-specific information about assets 1 and 2 but they do not have any factor information. Investor group C has asset-specific information about asset 3 and information about factor 1. Investor group D has asset-specific information about asset 4 and information about factor 2. Both factors f_1 and f_2 affect asset returns.

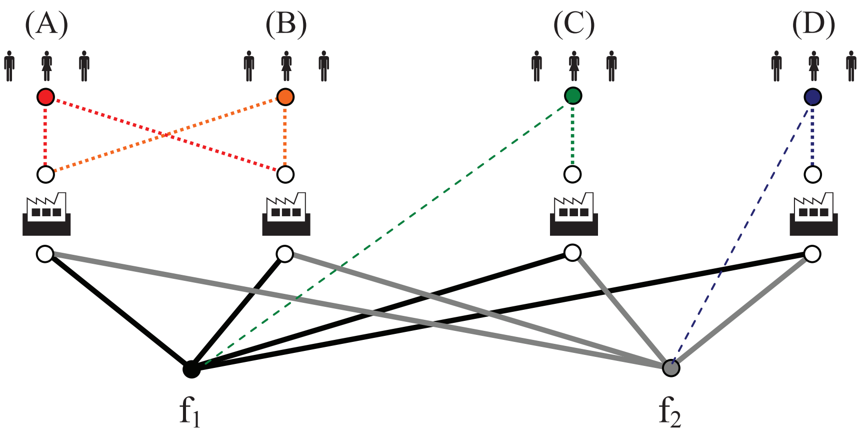


Figure 2

Closed-Form Solutions and Price Discounts

The graph depicts different market capitalizations associated with the 26 information structures we model. The relations between market capitalization and degree of frictions is defined to be linear and shown in Table 3. There are 22 structures that meet Section 4.1's assumptions needed for closed-form solutions and these are plotted with blue circles. The four structures that do not meet the assumptions needed for closed-form solutions are plotted with red diamonds

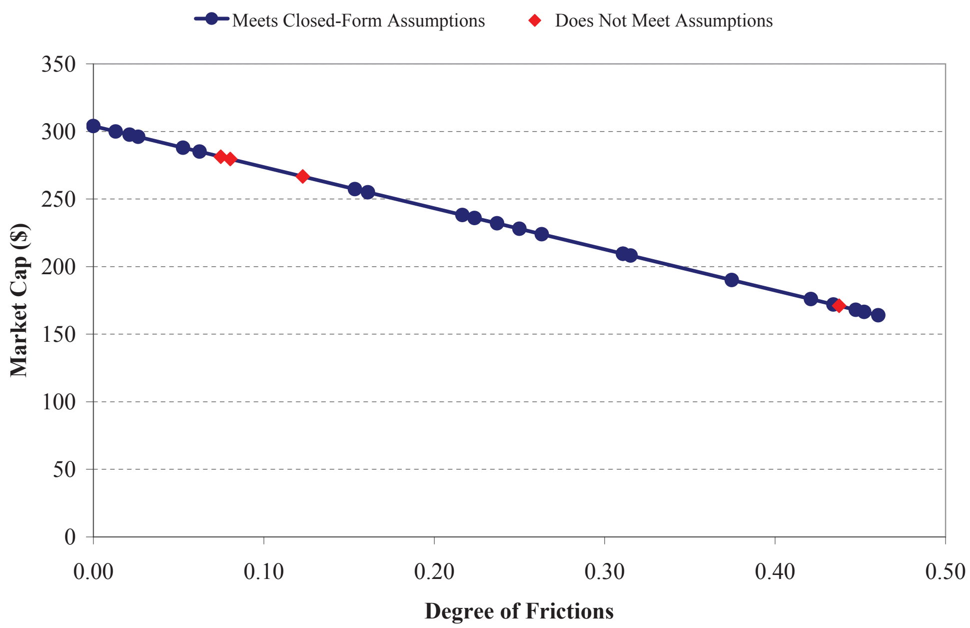


Figure 3

Closed-Form Solutions and Holdings Dispersion

Graphs depicts different levels of holdings dispersion associated with different degrees of frictions. Parameter $\mathbf{\Sigma}_\theta$ has the values 1.0 or 2.5 or 5.0 on its main diagonal. The matrix $\mathbf{B}$ is filled with the same values, either 0.5, 1.0 or 1.5. Holdings dispersion is defined as the root mean squared error (RMSE) or deviation from market portfolio weights. The RMSE is calculated across all assets and investors. Structures that meet Section 4.1's assumptions needed for closed-form solutions are plotted with blue circles. Structures that do not meet the assumptions needed for closed-form solutions are plotted with red diamonds.