Hong Kong University of Science and Technology FINA 584
Financial Modeling - Final Exam
Wednesday, December 16, 2009

Family Name: _____ **Familiar Name**: _____

HKUST Student ID#: _____

Directions: Please Read This Section First

A. You have three hours to complete this exam. This exam has been rated at 2 hours and 15 minutes for questions #1 to #4 so there should be plenty of time to do these problems. Do question #5 only if you have time because #5 is difficult and doesn't offer many points (think of it as like extra credit).

B. You may use the computer in front of you and one A4 sheet of paper with notes on it. Please do not consult friends, family, countrymen, or other classmates.

C. An Excel file has been uploaded for you. Right now, please change the name of the Excel file to:

    # # # # # # # # _ FamilyName_v01.XLS

    Where the first eight (8) digits are from your HKUST ID#. For example, the professor's filename might be "08812345_Seasholes_v01.XLS". You may not upload any other files onto the computer.

D. At the end of this exam, you are responsible for copying your Excel file to the professor's USB key. Both the professor and the TA will have USB keys.

E. Please put your HKUST ID# on the top of every page. Sorry, but we cannot be responsible for lost, mangled, unlabeled, orphaned, unreadable, or otherwise unintelligible pages.

F. Make sure you get to every question. The suggested time is written clearly at the top of each question. If you find yourself taking longer than this time, you will be rushed at the end. Points are roughly proportional to the suggested number of minutes for each question. Remember, this exam is graded on a curve.

G. You **must write** all your answers in the spaces provided. We grade answers on these pages.

H. You will be asked to do work on the computer. It is your responsibility to make sure the work is clearly presented. **Formatting is graded**. For example, please do work for question #2 on the worksheet titled "Q2". The grader is not responsible for hunting down obscure answers that may or may not be scattered in a worksheet.

I. Some questions towards the end of this exam are challenging. We know that. The exam will be scaled appropriately when grading.

J. The exam consists of pages numbered XXX to XXX. The last question is #XXX. The space provided should be sufficient to efficiently present the answer needed.

K. Please do NOT read too deeply into the questions. This is not an exam to test your memorization nor is it an exam that focus on minutia. Our goal is to test broad concepts, as best as possible, in a short-answer, computer-based, and numeric format.

[Page intentionally left blank]

HKUST ID#: _____

1. Warm-Up with Short Answer Questions (15 minutes; 15 total points)

A. What is the one-year forward rate from yr 3 to yr 4 given the zero coupon rates below? A. _____

(Please write your answer as a % with two decimal places.)

Term (yrs)	Rate (EAR)
1.0	2.62%
2.0	2.74%
3.0	2.89%
4.0	2.99%
5.0	3.15%
6.0	3.35%
7.0	4.51%
8.0	4.78%

B. What is the <u>two-year</u> forward rate from yr 5 to yr 7 given the zero coupon rates above? Make sure to <u>annualize</u> the rate. B. _____

(Please write your answer as a % with two decimal places.)

C. Monopolist's profit maximization. How many <u>units</u> should a firm produce given the numbers below: C. _____

(Please write your answer as a number with two decimal places.)

q = number of units produced (fractions ok)

p = price per unit

c = cost per unit

$p = 250 - 8.5 * q$

$c = 62.75$

_____ out of 15 points

2. Index Replication (45 minutes; 45 total points)

The goal of this problem is to replicate an index. Suppose you cannot trade the index directly (e.g., you are an American investor who can't easily trade PRC A-shares). You can, however, trade other securities such as ETFs. Some of the tradable securities are listed in the USA and some are listed in Hong Kong. We want you to combine the tradable securities into a portfolio that best replicates the daily returns of the non-tradable A-share index.

A. Values in USD

Convert all indices to USD values. We have provided exchange rates for you.

B. Daily Returns (of index and ETFs in USD)

Calculate the daily return of each index. Please use daily nominal returns and not daily log (CCR) returns. Make sure to use USD values. So we can check your answers, please fill in the average daily returns and standard deviations below.

(Please write your answers as %'s with two decimal places.)

	Non Tradable	Tradable ETFs				
	SHANGHAI A SHARE INDEX	ISHARES MSCI China INDEX	ISHARES S&P 500	ISHARES Russell 2000	HANG SENG INDEX ETF	ISHARES FTSE/ XINHUA A50
Mean	_____%	_____%	_____%	_____%	_____%	_____%
Stdev	_____%	_____%	_____%	_____%	_____%	_____%

2. Index Replication (continued)

C. Weights

Choose the weights of the five tradable securities. No short positions. Remember these are weights. There is no option to buy bonds. There is no option to introduce leverage. Students must confine themselves only to the five tradable securities. This problem asks you to choose fixed weights implying the portfolio is rebalanced daily (to maintain the weights).

(Please write your answers as %'s with two decimal places.)

	ISHARES MSCI CHINA INDEX	ISHARES S&P 500	ISHARES Russell 2000	HANG SENG INDEX ETF	ISHARES FTSE/ XINHUA A50
Weight	_____ %	_____ %	_____ %	_____ %	_____ %

D. Tracking error.

What is the daily tracking error of the replicated index ? Di. _____

(Please write your answer as a % with two decimal places.)

As best your can, please annualize the tracking error: Dii. _____

(Please write your answer as a % with two decimal places.)

E. Formatting.

Formatting counts. Have you formatted your model and answers so the TA can follow them easily? Do you use consistent numbers of decimal places? Have you followed the color scheme ? Is your spreadsheet laid-out in a logical fashion?

_____ out of 45 points

3. Lifetime Value of a Customer (45 minutes; 45 total points)

This question is based on an actual problem faced by a high-tech company in Asia. The numbers have been changed for security reasons. The overall goal is to calculate the lifetime value of a customer. We will approach this problem in a series of steps.

We provide data for customers acquired in Dec-2008. The data show the number of customers who continue to use the company's services each month. In a given month, customers not using the services are said to have "dropped out".

The company acquires 175 new customers in Dec-2008. Of these, 160 generate profits for the company in Jan-2009 and 15 drop out without generating profits. In Feb-2009, 148 customers generate profits while 12 more customers drop out. By Feb-2009, the total number of customers who have dropped out is 27 (calculated as 27 = 15 + 12 total drops). Data is only through Jul-2009.

A. Survival function.

The first part of the problem is to estimate survival function associated with the customers. In Dec-2008, there are 175 customers or 100% of the initial sample. In Jan-2009, there 160 remaining customers or 91.43% of the initial sample (where 91.43% = 160/175). In Feb-2009, there are 148 remaining customers or 84.57% of the initial sample. The formula below best describes the fraction of remaining customers using services in a given month:

$$F = e^{-\lambda t}$$

"λ" is a decay rate. The decay rate is a positive number such as 2.0% or 4.7% or 6.1% etc.

"t" is the month number and is {0, 1, 2, 3, 4, … }

Your job is to estimate the decay rate. In other words, solve for the "λ" that best fits the data (the % of remaining customers.) Only consider data from months 1 to 7 for now. There is no need to worry about month zero since the value of 100% is fit by definition. Fit the fraction of remaining customers and not the number customers. For this question, do not take logs.

(Please write your answers as %'s with two decimal places.)

A. _____

B. Best fit.

With a minimum of words, please tell us what criterion you used for "best fit" in part A above.

_____

_____

3. Lifetime Value of a Customer (continued)

C. Projections.

Extend your results. Use the estimated decay rate to project number of remaining customers for months #1 to #24. Look at month #17 (May-2010) and answer the two questions below.

What fraction (%) of the original customers are expected to remain? Ci. _____

(Please write your answer as a % with two decimal places.)

What is the number of remaining customers? Cii. _____

(Please write your answer as a number with two decimal places. Assuming a fractional number of customers is fine—do not convert to integers numbers.)

D. Average life.

The average tenure (life) of a customer is given by $1 / \lambda$ months.

Use the estimated λ. What is the average tenure of a customer? D. _____

(Please write your answer as a number with two decimal places.)

E. Half-life.

Half of the original customers are expected to remain in month #____, given by the following formula:

$$\text{Half-life} = \frac{\ln(2)}{\lambda}$$

This is also quite straightforward. What is the half-life in months ? E. _____

(Please write your answer as a number with two decimal places. You can use your projections from part C to check whether this answer makes sense. In the formula above, "ln" represents the natural logarithm.)

3. Lifetime Value of a Customer (continued)

F. Lifetime value of a customer.

The average customer is expected to generate profits of $35.72 per month.
What is the lifetime value of the average customer? F. _____

(Please write your answer as a number with two decimal places. Assume $35.72 is in today's dollars. We can forego discounting if the real rate is close to 0%.)

G. Weighted average life.

Confirm the average tenure estimated in part D of this problem. To do this, please estimate the weighted average life of a customer. The probability that a given customer has a tenure of "t" months is given by the probability function (PDF):

$$PDF = \lambda e^{-\lambda t}$$

Extend your projections to month #100. Now calculate the weighted average life of customers. Use the value of λ that you estimated earlier.

(Please write your answer as a number with two decimal places.)

G. _____

(Note: We know that the above PDF is for a continuous distribution. We want you to apply it to discrete data. Doing this will make your job easy. Simply assume the probability of a customer having a tenure of "t" months is given by the formula above. Do not worry about non-integer months (i.e., only focus on t = 0, 1, 2, 3, … , 100).

H. Formatting.

Formatting counts. Have you formatted your model and answers so the TA can follow them easily? Do you use consistent numbers of decimal places? Have you followed the color scheme ? Is your spreadsheet laid-out in a logical fashion?

_____ out of XX points

4. Step-Up Option (30 minutes; 30 total points)

Simulate monthly stock prices in order to value a variety of options. Please make sure to simulate monthly stock prices even if this may not be the most efficient frequency. The stock does not pay dividends. Here is the data you need:

Inputs	Value
P0	42.00
rf (CCR)	4.00%
Sigma (CCR)	18.00%

A. What are the risk-neutral parameters needed to simulate this stock price?

(Please write your answers as a %'s with four decimal places.)

Monthly drift Ai. _____

Monthly volatility Aii. _____

B. What is the value of a European call option ? B. _____

(Please write your answer as a number with two decimal places. This is a two-year "at-the-money" option.)

Inputs	Value
X	42.00
T (yrs)	2.00

C. What is the value of a "step-up" European-type call option with the following payoff structure: C. _____

(Please write your answer as a number with two decimal places.)

- If the stock finishes the two years less than $42, the option pays $0.00

or

- If the stock finishes the two years [$42, $45) the option pays $2.00

or

- If the stock finishes the two years [$45, $50), the option pays $4.00

or

- If the stock finishes the two years [$50, ∞), the option pays $16.00

D. Formatting

_____ out of XX points

5. Basket Option (7 total points)

Compare the value of an option written on a single stock to the value of an option written on a basket (portfolio) containing three, very similar stocks.

Inputs	**Stock #1**	**Stock #2**	**Stock #3**
P0	42.00	42.00	42.00
Sigma (CCR)	18.00%	18.00%	18.00%
rf (CCR) = 4.00%			

The basket consists of a 1/3rd position in each of three stocks. Since the stocks have the same, current price, the basket will consist of 1/3 share of each. To simulate correlated stock prices, you need the correlation matrix:

	Stock #1	**Stock #2**	**Stock #3**
Stock #1	1.00	0.50	0.30
Stock #2	0.50	1.00	0.10
Stock #3	0.30	0.10	1.00

Next, we "factor" the correlations matrix and get a lower-triangular matrix "D".

	Stock #1	**Stock #2**	**Stock #3**
Stock #1	1.0000	0.0000	0.0000
Stock #2	0.5000	0.8660	0.0000
Stock #3	0.3000	-0.0577	0.9522

To simulate the three stock prices:

i. Draw three (3) uniform random numbers each month: U[0,1]

ii. Convert the uniform random numbers to standard normal random numbers: N[0,1]
Call theses { X_1, X_2, X_3 }

iii. You need to convert the standard normals to the shock terms. You need to use matrix math. Each month, you can put the variables { X_1, X_2, X_3 } in either a column vector or a row vector. It's up to you. If the standard normals in month "t" are in a column vector ($X_{C,t}$), then the column vector with the three shock terms ($\varepsilon_{C,t}$) is given by: $\varepsilon_{C,t} = D\, X_{C,t}$ (below left). If the standard normals are in a row vector (X_R), then the row vector with the three shock terms (ε_R) is given by: $\varepsilon_{R,t} = X_{R,t}\, D^T$ and shown below right. The "T" superscript is "Transpose". Note that $X_R = X^T_C$ and $\varepsilon_R = \varepsilon^T_C$.

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.50 & 0.87 & 0.00 \\ 0.30 & -0.06 & 0.95 \end{bmatrix} \cdot \begin{bmatrix} X_{1,t} \\ X_{2,t} \\ X_{3,t} \end{bmatrix} \qquad \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{3,t} \end{bmatrix} = \begin{bmatrix} X_{1,t} & X_{2,t} & X_{3,t} \end{bmatrix} \cdot \begin{bmatrix} 1.00 & 0.50 & 0.30 \\ 0.00 & 0.87 & -0.06 \\ 0.00 & 0.00 & 0.95 \end{bmatrix}$$

A. What is the value of a two-year, at-the-money, European call option written on the basket ? A. _____

(Hint: Should the answer to this section be more or less than your answer to Question 4B ?)

_____ out of XX points

Grading (for official use only)

Question 1A	_____	5 points	Expected time is 15 min
Question 1B	_____	5 points	
Question 1C	_____	5 points	
Question 2A	_____	5 points	Expected time is 45 min
Question 2B	_____	10 points	
Question 2C	_____	10 points	
Question 2Di	_____	5 points	
Question 2Dii	_____	5 points	
Question 2E	_____	10 points	
Question 3A	_____	10 points	Expected time is 45 min
Question 3B	_____	5 points	
Question 3Ci	_____	2 points	
Question 3Cii	_____	3 points	
Question 3D	_____	5 points	
Question 3E	_____	2 points	
Question 3F	_____	3 points	
Question 3G	_____	7 points	
Question 3H	_____	8 points	
Question 4Ai	_____	5 points	Expected time is 30 min
Question 4Aii	_____	5 points	
Question 4B	_____	5 points	
Question 4C	_____	10 points	
Question 4D	_____	5 points	
Question 5A	_____	7 points	Do only if you have time

Total [ ] 142 points

[Page intentionally left blank]