

**1. Warm-Up with Short Answer Questions (15 minutes; 15 total points)**

**A. What is the one-year forward rate from yr 3 to yr 4 given the zero coupon rates below?**

**A. 3.29% (5 pts)**

Term	Rate	Fwd 1yr	Fwd 2yr
1	2.62%	2.62%	
2	2.74%	2.86%	2.74%
3	2.89%	3.19%	3.03%
4	2.99%	3.29%	3.24%
5	3.15%	3.79%	3.54%
6	3.35%	4.36%	4.07%
7	4.51%	11.75%	7.99%
8	4.78%	6.69%	9.19%

- 4 pts if reporting forward rate from yr 2 to yr3
- 2 pts if using log rate

-5 pts if in other cases where the wrong formula is used

$$(1 + f_{3,4})^1 = \frac{(1 + r_4)^4}{(1 + r_3)^3}$$

**B. What is the two-year forward rate from yr 5 to yr 7 given the zero coupon rates above? Make sure to annualize the rate.**

**B. 7.99% (5 pts)**

- 3 pts if you put 16.62% (i.e, you forgot to annualize)
- 3 pts if annualize twice
- 2 pts if annualize by dividing 16.62% by 2, or dividing 16.62% by 12
- 5 pts if in other cases where the wrong formula is used

$$(1 + f_{5,7})^2 = \frac{(1 + r_7)^7}{(1 + r_5)^5}$$

**C. Monopolist's profit maximization. How many units should a firm produce given the numbers below:**

**C. 11.01 (5 pts)**

This problem can be done quickly using solver. Just maximize profit by adjusting the number of units. Alternatively, you can always use calculus.

q	11.01
p	156.38
Rev	1722.42
Cost per q	62.75
Tot Cost	691.17
Profit	1031.25

- 2 pts if wrongly set p=250+8.5q in Excel
- 3 pts if q=187.25/8.5
- 3 pts if ignoring costs when calculating profits
- 5 pts in other cases when the results are wrong

## 2. Index Replication (45 minutes; 45 total points)

This problem is a form of Sharpe (1992) style analysis. The A-Share index plays the role of the mutual fund. The tradable securities play the role of the risk factors. The goal remains to find the weights to best replicate the daily returns.

One goal of this problem is to test if you recognize Sharpe (1992) style analysis when it is in a form that is slightly different from what we did in class. We did not want to give you a problem where you simply repeated work already done as part of our assignments.

### A. Values in USD (5 pts)

Assumed correct if Part B below is correct  
We put the data in a form that made converting currencies straightforward.

### B. Daily Returns (of index and ETFs in USD) (10 pts)

	Non Tradable	Tradable ETFs				
	SHANGHAI A SHARE INDEX	ISHARES MSCI China INDEX	ISHARES S&P 500	ISHARES Russell 2000	HANG SENG INDEX ETF	ISHARES FTSE/ XINHUA A50
Mean	0.10%	0.11%	0.01%	0.02%	0.06%	0.13%
Stdev	1.99%	2.55%	1.46%	1.83%	1.91%	2.63%

- 5 pts if wrong daily return formula is used
- 5 pts for wrong mean
- 5 pts for not using USD dollar indices

### C. Weights (10 pts)

	ISHARES MSCI CHINA INDEX	ISHARES S&P 500	ISHARES Russell 2000	HANG SENG INDEX ETF	ISHARES FTSE/ XINHUA A50
Weight	0.00 %	28.13 %	0.00 %	9.32 %	62.55 %

- 3 pts for calculating tracking error in the wrong way
- 5 pts for not minimizing tracking error or SSE
- 5 pts for minimizing std of replicated index returns
- 5 pts if weights don't sum to 100%
- 5 pts if some weights are negative
- 8 pts if solver not used
- 8 pts if no answer, but try to set up the solver

### D. Tracking error.

**What is the daily tracking error of the replicated index ?** **Di.** **1.52% (5 pts)**

- 3 pts for using "beta" to get the fitted return
- 3 pts for reporting std of the replicated index return
- 5 pts for no answer
- 2 pts for reporting variance of the tracking errors
- 2 pts for reporting regression standard error

**As best your can, please annualize the tracking error:** **Dii.** **24.05% (5 pts)**

- 2 pts for using 12<sup>th</sup> power
- 2 pts for multiplying by 365
- 3 pts for dividing by 250
- 3 pts for calculating the std of "annual" returns' tracking errors

### E. Formatting.

Formatting is generally graded as following (10 pts)

Neatness	+3
Consistency e.g., having consistent number of decimals	+3
Using the color scheme from class this includes NOT embedding input numbers in cells in such a way as to hide them	+3
Other	+1

### 3. Lifetime Value of a Customer (45 minutes; 45 total points)

**A. Survival function.**

Your job is to estimate the decay rate.

Some students estimated geometric decay rates (similar to discount rates) rather than use the exponential decay given in the question.

**A. 8.14% (10 pts)**

- 2 pts for expressing in wrong percentage form
- 2 pts for minimizing the sum of tracking errors
- 4 pts for running regressions to get lambda (can get a geometric rate)
- 4 pts for not minimizing tracking error or SSE
- 5 pts for using moving average
- 5 pts for estimating average geometric decay
- 5 pts for averaging the decay rate of the 7 periods
- 5 pts for not using all the 7 periods' data to estimate lambda
- 8 pts for not using lambda to fit the decay rate
- +2 pts for a good try

**B. Best fit.**

**(5 pts)**

Minimize least squares

Alternative: minimize the variance of the errors

- +3 pts for a good try

**C. Projections.**

What fraction (%) of the original customers remain?

**Ci. 25.06% (2 pts)**

- 1 pts for using geometric decay

What is the number of remaining customers?

**Cii. 43.85 (3 pts)**

- 1 pts for using geometric decay

**D. Average life.**

Use the estimated  $\lambda$ . What is the average tenure of a customer?

**D. 12.28 (5 pts)**

- 5 pts for not using  $1/\lambda$

**E. Half-life.**

What is the half-life in months ?

**E. 8.51 (2 pts)**

**F. Lifetime value of a customer.**

What is the lifetime value of the average customer?

**F. 438.73 (3 pts)**

- 2 pts for using half life

**G. Weighted average life.**

Now calculate the weighted average life of customers.

**G. 12.24 (7 pts)**

- 7 pts for using one period data only
- 4 pts for multiply by # of customers, not months
- 4 pts for summing up pdf/wrong weighted tenure
- 7 pts for reporting average pdf

**H. Formatting.**

Formatting is generally graded as following

**(8 pts)**

- Neatness +2
- Consistency +2
  - e.g., having consistent number of decimals
- Using the color scheme from class +2
  - this includes NOT embedding input numbers in cells in such a way as to hide them
- Other +2

**4. Step-Up Option (30 minutes; 30 total points)**

Many students used two-month long periods rather than a monthly frequency.

**A. What are the risk-neutral parameters?**

Monthly drift **Ai. 0.1983% (5 pts)**

Monthly volatility **Aii. 5.1962% (5 pts)**

- 2 pts if not in percentage form
- 2 pts if  $dt=0.1667$
- 3 pts if volatility formula is  $\sigma/\sqrt{dt}$
- 5 pts if drift formula is  $rf/12$

**B. What is the value of a European call option ?**

**B. 5.90 (5 pts)**

- 1 pts if the result is too far from 5.90
- 2 pts if stock price formula is wrong
- 2 pts if using wrong discount rate
- 2 pts if wrong  $dt$
- 5 pts for using Black-Scholes formula

**C. What is the value of a “step-up” European-type call option with the following payoff structure:**

**C. 5.40 (10 pts)**

- 10 pts for not using simulation
- 7 pts if step option payoff is wrong

**D. Formatting**

**Formatting is generally graded as following**

**(5 pts)**

- |  |    |
|--|----|
| Neatness   | +1 |
| Consistency<br>e.g., having consistent number of decimals  | +1 |
| Using the color scheme from class<br>this includes NOT embedding input numbers<br>in cells in such a way as to hide them | +1 |
| Other  | +2 |

**5. Basket Option (7 total points)**

**A. What is the value of a two-year, at-the-money, European call option written on the basket ?**

**A. 4.85 (7 pts)**

- 2 pts if  $dt$  is wrong
- 3 pts for wrong use of covariance matrix
- 2 pts for wrong payoff formula of the option
- 7 pts for using the answer in 4.B

Some students didn't transpose the "D" matrix when using row vectors.