1. Warm-Up with Short Answer Questions (15 minutes; 15 total points)

A. What is the one-year forward rate from yr5 to yr6 given
The zero coupon rates below?

Term	Rate
1.0	2.1%
2.0	2.3%
3.0	2.5%
4.0	2.8%
5.0	3.2%
6.0	3.9%
7.0	4.5%

Q1A: ___________

B. Which of the following is a necessary input to the
Black-Derman-Toy model (Check all that apply):

- ☐ The coupon yield curve
- ☐ The zero-coupon yield curve
- ☐ The volatilities of coupon rates
- ☐ The volatilities of zero-coupon bonds
- ☐ The covariance matrix of zero-coupon bonds
- ☐ The relative amounts (weights) of zero-coupon bonds
- ☐ The amount of riskfree assets in the economy
- ☐ The weight of the riskfree asset in the economy
- ☐ The expected return on the market portfolio

Q1B: ___________

C. Use goal seek to find "*x*" such that:

$$0.70x^4 - 0.50x^3 + 0.30x^2 + 0.30x - 0.05 = 0$$

Q1C: ___________

___________ out of 15 points

2. Outperformance Options (55 minutes; 55 total points)

This question refers to an at-the-money European-type call option. As in class, I'm thinking of a typical stock that trades in Hong Kong. Let's consider HSBC. You believe the worst of the recession is over. You believe HSBC's stock price is still quite cheap. You expect HSBC to go up 20% over the next year. Therefore, you are thinking of buying a call option with the following parameters.

$S_0(HSBC)$ = \$65.00
$X(HSBC)$ = \$65.00
$\tau(HSBC)$ = 1.00 year
$\sigma(HSBC)$ = 32.00%
r_f = 1.00%

Notes: The riskfree rate (r_f) and volatility (σ) are quoted as continuously compounded annual rates. The dividend rate is assumed to be zero for simplicity.

A. Given the information above, simulate HSBC's stock price at a <u>monthly</u> frequency. Use Monte Carlo analysis and your simulated stock price to estimate the price of the European call option described above. You should run at least 10,000 simulations. Write your answer in the box. Save the output from the Monte Carlo in a worksheet named "2A".

Q2A: Value of Call(HSBC) ___________
10 points

B. Suppose there is an Exchange Traded Fund (ETF) which mimics the return of the Hong Kong market index. By chance, the ETF is also trading at \$65.00 today. You believe the market will not recover as quickly as the individual stock above. You expect the ETF to go up only 10% over the next year. Finally, the ETF is expected to be <u>less</u> volatile than the above stock.

$S_0(ETF)$ = \$65.00
$X(ETF)$ = \$65.00
$\tau(ETF)$ = 1.00 year
$\sigma(ETF)$ = 18.00%
r_f = 1.00%

Given the information above, simulate the ETF's price at a <u>monthly</u> frequency. Use Monte Carlo analysis and your simulated stock price to estimate the price of a European call option. You should run at least 10,000 simulations. Write your answer in the box. Save the output from the Monte Carlo in a worksheet named "2B".

Q2B: Value of Call(ETF) ___________
5 points

C. How much is an outperformance call option on HSBC (vs. the ETF) worth? If the price of HSBC is greater than the price of the ETF in one year, the outperformance option pays the difference between HSBC's price and the ETF's price. If the price of HSBC is less than or equal to the price of the ETF in one year, the outperformance option pays zero

Assume the two stock prices are uncorrelated. You should run at least 10,000 simulations. Write your answer in the box. Save the output from the Monte Carlo in a worksheet named "2C".

> Q2C: Value of Outperf Option ___________
> 15 points

D. Assume that HSBC's returns and the ETFs returns have a 0.58 correlation coefficient. Now recalculate how much the outperformance call option on HSBC (vs. the ETF) is worth. Write your answer in the box. Save the output from the Monte Carlo in a worksheet named "2D".

To simulate two correlated returns, we need to simulate two simulate "shocks". The tilde (or "~") above a variable signifies that this is a random variable. We will call these shocks "$\tilde{\varepsilon}_1$" and "$\tilde{\varepsilon}_2$". We will call the correlation coefficient $\rho = 0.58$. If $\tilde{x}_1$ and $\tilde{x}_2$ are independent standard normal variables, then $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are the correlated normal variables. The first normal is $\tilde{\varepsilon}_1 = \tilde{x}_1$ The second normal variable is $\tilde{\varepsilon}_2 = \rho \cdot \tilde{x}_1 + \tilde{x}_2 \sqrt{1-\rho^2}$

> Q2D: Outperf. Opt w/ $\rho = 0.58$ ___________
> 15 points

E. Format your spreadsheet so that the HSBC and ETF inputs are clearly displayed.
- Add a graph of the monthly stock prices (do this after doing the simulations)
- You do not have to automate the Monte Carlo analysis
- Consider pointing to the correct cells in worksheets "2A", "2B", "2C", and "2D"
- If you have time, make the worksheet look as nice as possible

> Q2E: ___________
> 10 points

Learning point: **Think about why most CEOs prefer traditional options on their own company's stock rather than outperformance options.**

> ___________ out of 55 points

3. Bond Portfolio Optimization (50 minutes; 50 total points)

It is December 31, 2008. You are advising the CFO of a large company. The CFO needs to raise €2,000,000 and plans to issue bonds. The market is ready to possibly buy three types coupon bonds with terms of 5yrs, 10yrs, and/or 20yrs. The CFO's problem is related to a portfolio of bonds. He must choose how much weight to put on each of the three bond types.

The CFO is attune to modern risk-management and wants a 8.5 duration of the bond portfolio. The value is chosen to match the duration of the assets in his firm. In this problem, we are talking about the Macaulay modified duration of bonds—a measure that is related to, but different from, a bond's weighted average life.

The terms of the bonds are shown below. The next page shows a screenshot of how we (strongly) suggest you set up this problem. It contains some very helpful information. Make sure Excel's "Analysis Toopak" has been added.

	Bond(i)	**Bond(ii)**	**Bond(iii)**
Settlement	**31-Dec-2008**	**31-Dec-2008**	**31-Dec-2008**
Maturity	**31-Dec-2013**	**31-Dec-2018**	**31-Dec-2028**
Coupon	**3.00%**	**5.00%**	**6.00%**
YTM	**2.80%**	**5.40%**	**5.85%**
Face	**100.00**	**100.00**	**100.00**
Freq	**1**	**1**	**1**
Basis	**1**	**1**	**1**

A. Use Excel's "PRICE" function to find the price per bond for each of the three bonds. As shown on the next page, the price of the first bond has been calculated for you.

Q3A:	Price of Bond(i)	100.9212
	Price of Bond(ii)	__________
	Price of Bond(iii)	__________
	10 points	

B. Use Excel's "DURATION" function to find each bond's weighted average life (WAL). Do not confuse this value with the modified duration. As shown on the next page, the weighted average life of the first bond has been calculated for you.

Q3B:	Wgt Avg Life of Bond(i)	4.7186
	Wgt Avg Life of Bond(ii)	__________
	Wgt Avg Life of Bond(iii)	__________
	5 points	

	Bond(i)	Bond(ii)	Bond(iii)		Total
Num of Bonds	19,817.44	0	0		
Value of Bonds	2,000,000				2,000,000
Portfolio Wgt	100%				
Wgt * MDur					
Price per bond	100.9212				
Duration (WAL)	4.7186				
Mod Duration	4.5901				
Settlement	31-Dec-2008	31-Dec-2008	31-Dec-2008		
Maturity	31-Dec-2013	31-Dec-2018	31-Dec-2028		
Coupon	3.00%	5.00%	6.00%		
YTM	2.80%	5.40%	5.85%		
Face	100.00	100.00	100.00		
Freq	1	1	1		
Basis	1	1	1		
					All-In
					YTM
Year	**i**	**ii**	**iii**		**Total**
0	2,000,000				
1	-59,452				
2	-59,452				
3	-59,452				
4	-59,452				
5	-2,041,197				
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

C. Use Excel's "mduration" function to find each bond's modified duration. This is the value we want. As shown on the next page, the modified duration of the first bond has been calculated for you.

Note: the mod. duration of a portfolio is the wgt average of the individual mod. durations.

Q3C:	Mod Duration Bond(i)	4.5901
	Mod Duration Bond(ii)	__________
	Mod Duration Bond(iii)	__________
	5 points	

D. Assume you hold 7,019.06 of Bond(i). Since the price is 100.9212 per bond, this position has a value of €708,373 = 7,019.06×100.9212. Assume you hold 6,500 of Bond(ii). Assume you hold 6,500 of Bond(iii). Major hint: if your bond prices are correct, the total portfolio value should be €2,000,000.

Use the prices and the coupon information to write down the total cashflows associated with holdings of each type of bond. The screenshot on the previous page does this except the holdings are 19,817.44 of Bond(i). Sum the cashflows to get the total cashflows associated with the project. Now use the total cashflows to calculate the "All-In YTM". This is the CEO's average cost of borrowing

Q3D: All-In YMT __________
15 points

E. Figure out how the CEO can achieve the lowest cost of all-in borrowing. Use Excel's Solver to choose how many of each bond to hold in your portfolio. Fractional values are OK. The company must rate €2,000,000. The bonds must have a modified duration of 8.5:

Note that if you only have 19,817.44 of Bond(i), you would:
- Raise €2,000,000 (good)
- Have an all-in YTM of 2.80% (good, the YTM is low)
- Have a modified duration of 4.5901 (BAD, the mod. duration is below the 8.5 needed)

Q3E:	Number of Bonds(i)	__________
	Number of Bonds(ii)	__________
	Number of Bonds(iii)	__________
	15 points	

__________ out of 50 points

4. Do only if you have time (5 points)

Use Monte Carol analysis to estimate the value of π (called "pi"). We suggest you follow these steps:

A. Look at the "Target" below. It is a Cartesian coordinate system (or "X-Y" axis). Each block is ½×½. Five pairs of X-Y points are marked on the target. The point (0,0) is called the "origin".

B. Draw a square centered at the origin. Make sure the square has length two ($L=2$) on each side.

C. Sketch a circle centered at the origin. Make the radius of the circle equal to one ($r=1$) so the diameter is two ($d=2r=2$). If you have done this correctly, the circle will just fit in the square.

D. The equation of the circle is: $X^2 + Y^2 = 1$
This means that any pair of random numbers {X,Y} will be inside the circle if: $X^2 + Y^2 \le 1$

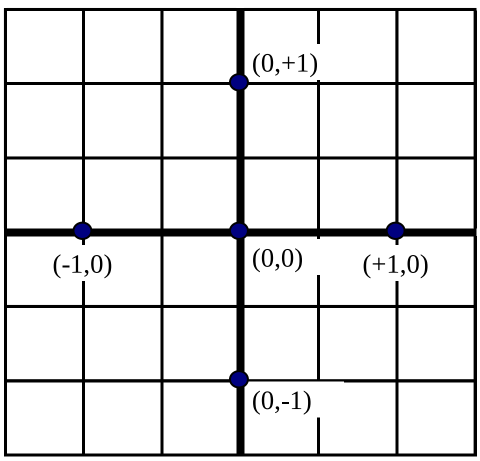
E. Draw random numbers to simulate "throwing darts" are the target. Make a note if a dart lands in the square, in the circle, in both, or in neither. t the number of darts that land in different areas. These counts, along with the formulas beCounlow, can help you estimate the value of π.

$$Area(Square) = L^2 \qquad\qquad Area(Circle) = \pi r^2$$

Put your answer in the space provided. Save any results from the Monte Carol analysis in a spreadsheet titled "4A"

My estimated value of π is: ___________________

The Target



__________ out of 5 points